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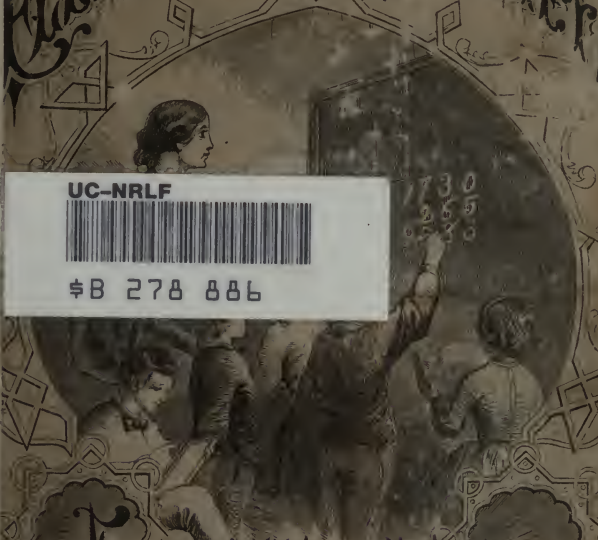
ELEMENTARY ARITHMETIC



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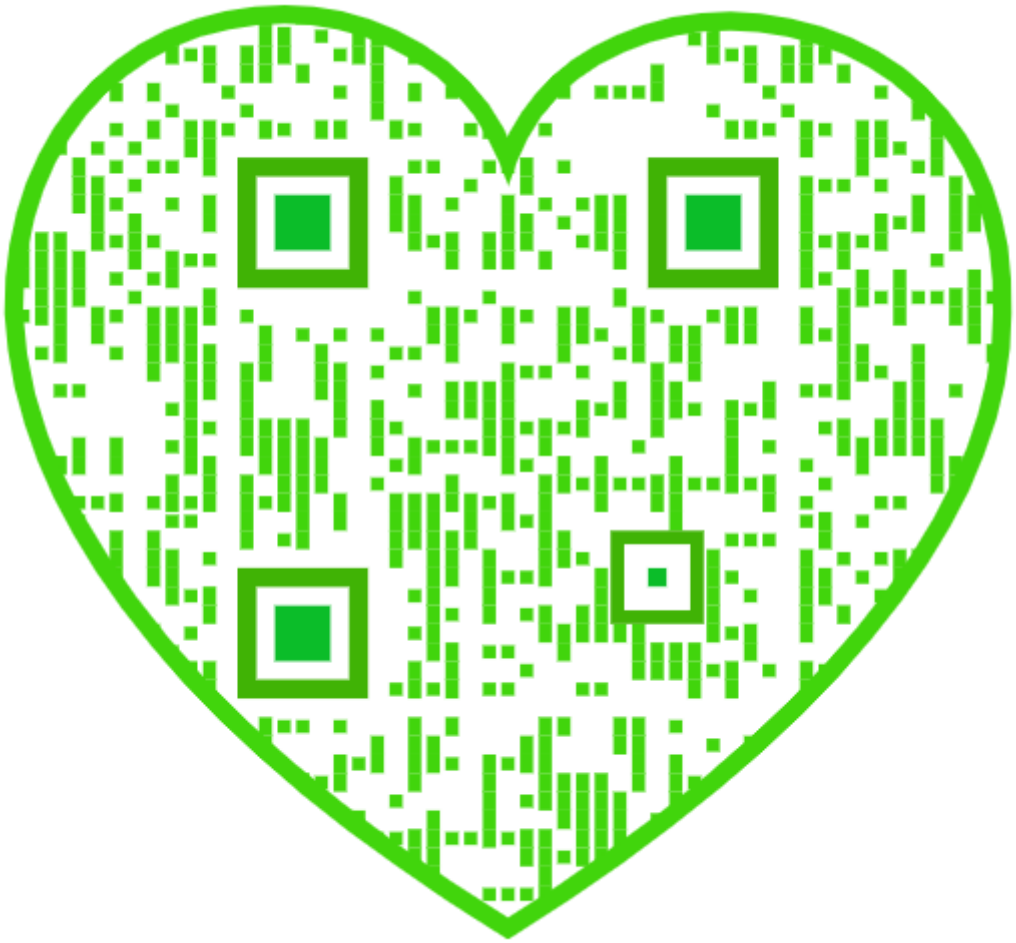

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FRENCH'S MATHEMATICAL SERIES.

ELEMENTARY ARITHMETIC,

FOR THE SLATE;

IN WHICH

METHODS AND RULES

ARE BASED UPON

PRINCIPLES ESTABLISHED BY INDUCTION.

BY

JOHN H. FRENCH, LL.D.

If Principles are understood, Rules are useless.

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NEW YORK:
HARPER & BROTHERS.

1869.

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PUBLISHERS' NOTICE.

FRENCH'S ARITHMETICS.

This Series consists of Five Books, viz.:

I.—FIRST LESSONS IN NUMBERS.
II.—ELEMENTARY ARITHMETIC.
III.—MENTAL ARITHMETIC.....In Press.
IV.—COMMON SCHOOL ARITHMETIC..
V.—ACADEMIC ARITHMETIC.....In Preparation.

The Publishers present this Series of Text-Books to American Teachers, fully believing that they contain many new and valuable features that will especially commend them to the *practical* wants of the age.

The plan for the Series, and for each book embraced in it, was fully matured before any one of the Series was completed; and as it is based upon true philosophical principles, there is a harmony, a fitness, and a real progressiveness in the books that are not found in any other Series of Arithmetics published.

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PREFACE.

THE object of this book, designed especially for beginners in Written Arithmetic, is twofold, viz.: 1*st*. To give to young learners a good foundation for the study of the Science of Numbers, by basing all Methods of Operation upon Principles; and, 2*d*. To give them as much knowledge as possible of the business affairs of life, by the introduction of business transactions stated in correct business language.

The plan of the work differs, in most of its essential points, from that of other works of a like grade. To these points of difference—and it is confidently believed of superiority—the attention of parents and teachers is particularly invited.

General Divisions.—Chapters.—The work is divided into six chapters, the first one of which is devoted to Integers; the second, to Decimals; the third, to Compound Numbers; the fourth, to Fractions; the fifth, to Percentage, and the sixth, to Miscellaneous Review Problems.

Integers and Decimals are but parts of the same class of numbers, the latter being an extension of the decimal scale to the right of the decimal point, or below ones. They are both subject to the same laws, and all operations upon them are based upon the same principles. Therefore, in the natural order of arrangement of subjects, the proper place for Decimals is immediately after Integers.

Compound Numbers differ from Integers and Decimals only in the scales of increase and decrease, which, in the latter, are uniform and decimal, while in the former, they are irregular and varying. The new facts to be learned in Compound Numbers are, the scales or tables, and their application to the processes of Addition, Subtraction, Multiplication, and Division. Only two of the denominations given in this book—$5\frac{1}{2}$ or 5.5 yards are 1 rod, and $30\frac{1}{4}$ or 30.25 square yards are 1 square rod—are Mixed Numbers, and these are as well expressed decimally as fractionally. There is, therefore, no good reason why Fractions should precede Compound Numbers, no knowledge of the former being necessary in studying the latter, while the advantages of the reverse order of arrangement are obvious.

A knowledge of the preceding chapters prepares the pupil, on reaching Fractions, to comprehend the new facts to be learned, viz.: the Notation of Fractions, the General Principles of this class of numbers, and the application of these principles to the operations upon Fractional Numbers.

In the general arrangement of the work, and also in its details, the fact has never been lost sight of, that only a small portion of all the children who commence the study of Arithmetic go through their text-book; and that a child should be taught first that which it is *most* desirable and important for him to know; so that, whenever he leaves school, the knowledge he has acquired will be of practical value to him in after life.

Subdivisions.—Sections and Cases.—Each chapter except the last is divided into sections, and wherever necessary, the sections are subdivided into cases. The subjects of corresponding sections in the first four chapters are similar. For example, Section I. in each chapter is Notation and Numeration; Section II. is Addition, etc. The cases in the several Sections correspond to each other, wherever the nature of the subjects will admit. Especial attention is invited to the following points in the several chapters and sections:

CHAPTER I.—The first Method of Addition will familiarize pupils with the reason for the "carrying process," and also accustom them to add the reserved tens of the sum of any column to the *first* figure of the next column, instead of the *last*.

All the cases in Multiplication and Division are based upon a few general principles, readily understood, and hence easily remembered.

Long Division precedes Short Division, because it is simpler. In the former, all the partial results—quotient figure, partial dividend, partial product, and partial remainder—are written down; while in the latter, the quotient figure only is written, the memory being taxed to form all the combinations, and retain all the other partial results in the process. Long Division is a general process, while Short Division is a contraction, limited in its application. The partial results written in the former are really *Visible Objects*, while in the latter they become *Abstract Numbers*. The natural order of mental development, *Perception before Memory*, has therefore been observed, in placing Long Division before Short Division.

The divisor is written at the right of the dividend. This arrangement is as convenient for Short Division as that of placing the divisor at the left of the dividend; while in Long Division, the quotient is written under the divisor, and the factors of the partial dividends are thus brought nearer together, and therefore in a more convenient position for multiplication.

A section embracing the simplest cases in Measurement is introduced into this chapter, because, 1*st*. The subject is interesting to children, and is readily understood by them as soon as they have passed over the fundamental rules; and, 2*d*. The cases here given are the basis of the objective method used in illustrating some of the principles of Decimals.

CHAPTER II.—The Diagram of Decimal Notation, the Table of Values of Decimal Numbers, and the Decimal Notation and Numeration Table, when thoroughly understood, give to pupils a clearer comprehension of Decimals than they can obtain without these aids.

The reason for placing the decimal point in the product in Multiplication is derived from the principles of Measurement; and that for placing the decimal point in Division, from a general principle of Division. These reasons are strictly philosophical, and easily understood, and are entirely independent of Fractions.

The divisions of the dollar being decimal, and all computations in U. S. money being based upon the same principles as Decimals, the subject of U. S. or Federal Money is embraced in this chapter, and the necessity for separate principles and rules is obviated.

CHAPTER III.—The Tables of Compound Numbers are arranged in the order in which they will be the most readily comprehended by young pupils; and only those denominations in actual use are given. A few tables, such as Troy Weight, Surveyors' Measure, etc., being of limited use, are omitted.

The Tables of the Metric System are given, because they are legalized by act of Congress; but they are not made prominent, because they are not yet in use.

CHAPTER IV.—The term *Similar Fractions* is used in place of *Fractions having a common denominator*. The simplicity and comprehensiveness of the term should secure its general adoption. The only important application of the subject, Common Multiple, is in the reduction of dissimilar to similar fractions. It is therefore presented in this chapter. The subjects of Least Common Multiple, and Common Divisors, not being essential to an elementary work, have been omitted.

The cases in Multiplication and Division are different from those in any similar work, and the methods are also new and superior. The method of Division is based upon the same general principle as is Division of Decimals.

The applications of Cancellation to Multiplication and Division are made the subject of a separate section.

CHAPTER V.—One general case is given, embracing all the general principles of Percentage; and to this case all the methods for computations in Insurance, Commission, Profit and Loss, Stocks, Banking, and Interest, are referred.

The method for Interest is new, and its simplicity, absolute accuracy, and general application, make it superior to any heretofore presented.

CHAPTER VI.—The problems in this chapter embrace applications of all the principles and methods of computation contained in the previous chapters of the book.

Inductions.—Each chapter, as well as many of the sections and cases, commences with Illustrations which form Visible Objects. Then follow, in the natural order, Concrete and Abstract Numbers.

Illustrations.—The cuts, maps, and diagrams, all of which are new, are intended not only to aid the pupil in acquiring a clear understanding of the subjects they illustrate, but also to educate his eye, cultivate his taste, and teach him some useful fact or principle.

Examples and Problems.—Care has been taken to use these terms—so often used indiscriminately—in accordance with their signification.

In the induction to a Case or Method, one or more examples are solved, and the solution is inductively explained. These examples, except in Currency, Compound Numbers, and Percentage, contain only abstract numbers, because a general principle should not be deduced from a special or limited application.

The problems are derived from actual business tranactions, and *are all new.* The facts stated in them have been obtained from reliable authorities, and the business transactions are in accordance with business customs.

Each chapter closes with a section of Review Problems, designed to test the pupil's knowledge of all the previous chapters.

Definitions.—The definitions, being intended for young minds, are stated in the inductive form. They are brief, accurate, and comprehensive.

Oral Exercises.—It is a conceded fact that children learn methods and processes of computation more readily than they learn combinations. Many persons go on through life performing all their numerical computations by counting. They never learn to step more than a *one* at once. The Oral Exercises, if practiced according to the directions given, will break up the counting plan—pupils will learn to step from given parts to required results without hesitation, and will soon become rapid and accurate computers.

Tables of Combinations.—The Addition, Subtraction, Multiplication, and Division tables are presented in a new, and, it is believed, a more attractive form than the solid pages of figures that have greeted the eyes of children from time immemorial.

Principles and Rules.—Principles are deduced immediately from the inductive examples, and are followed by problems which require the pupil to apply the principles. He is thus made familiar with reasons for the processes, before the rules are given; and whenever he applies a rule in solving a problem, the *why* is as familiar to him as the *how.* Rules without principles are soon forgotten; while if principles are understood, rules are useless.

Methods of Proof.—Self-reliance is one of the most important things that can be taught to children. To do their work correctly, and to feel sure that they are correct, they must be drilled in combinations until they add, subtract, multiply, and divide without making any mistakes. As methods of proof generally retard children in reaching this most desirable degree of accuracy, they have been omitted from this work.

Teachers' Manual.—The last twelve pages of the book are devoted to notes, remarks, suggestions, and hints to teachers; and to this Manual frequent references are made in the body of the work. Teachers should not fail to consult the Manual, whenever reference is made to it.

The many new and valuable features of the book, its superior typography and beautiful illustrations, its great number of practical problems drawn directly from business life, and its adaptation to the wants of graded schools, and to the capacity of beginners in schools of any grade, will, it is hoped, secure for it the attention and careful examination of intelligent, progressive teachers.

UNIVERSITY OF CALIFORNIA

CONTENTS.

CHAPTER I.—INTEGERS.

		PAGE
SECTION	I.—Notation and Numeration	9
SECTION	II.—Addition	17
SECTION	III.—Subtraction	31
SECTION	IV.—Multiplication	45
SECTION	V.—Division	64
SECTION	VI.—Measurements	83
SECTION	VII.—Problems in Integers	89

CHAPTER II.—DECIMALS.

SECTION	I.—Notation and Numeration	94
SECTION	II.—Addition	101
SECTION	III.—Subtraction	105
SECTION	IV.—Multiplication	108
SECTION	V.—Division	113
SECTION	VI.—United States Money	121
SECTION	VII.—Problems in Decimals	125

CHAPTER III.—COMPOUND NUMBERS.

SECTION	I.—Notation and Reduction	129
SECTION	II.—Addition	149
SECTION	III.—Subtraction	151
SECTION	IV.—Multiplication	155
SECTION	V.—Division	157
SECTION	VI.—Problems in Compound Numbers	159

CHAPTER IV.—FRACTIONS.

PAGE

SECTION I.—Induction and Notation 162
SECTION II.—Reduction 166
SECTION III.—Addition 171
SECTION IV.—Subtraction 174
SECTION V.—Multiplication 177
SECTION VI.—Division 181
SECTION VII.—Cancellation 184
SECTION VIII.—Problems in Fractions 186

CHAPTER V.—PERCENTAGE.

SECTION I.—Notation and Numeration 189
SECTION II.—General Applications 190
SECTION III.—Commission 191
SECTION IV.—Insurance 192
SECTION V.—Profit and Loss 193
SECTION VI.—Interest 195
SECTION VII.—Problems in Percentage 199

CHAPTER VI.

MISCELLANEOUS PROBLEMS 201

MANUAL FOR TEACHERS 209

OF THE UNIVERSITY OF CALIFORNIA

ELEMENTARY ARITHMETIC.

CHAPTER I.

INTEGERS.

SECTION I.

NOTATION AND NUMERATION.

1. In writing numbers, ten characters, called figures, are used.

The first figure, 0, is called a *cipher* or *naught*, and denotes nothing or the absence of number.

The other nine figures are used to represent the first nine numbers.

1. One.

2. Two.

3. Three.

4. Four.

5. Five.

6. Six.

7. Seven.

8. Eight.

9. Nine.

To express numbers greater than nine, two or more of these ten figures must be combined.

2. In writing numbers, every ten *ones* taken together are called a *ten*.

Ten is written					*10*
Two tens	are called	*twenty*,	written,		*20*
Three tens	" "	*thirty*,	"		*30*
Four tens	" "	*forty*,	"		*40*
Five tens	" "	*fifty*,	"		*50*
Six tens	" "	*sixty*,	"		*60*
Seven tens	" "	*seventy*,	"		*70*
Eight tens	" "	*eighty*,	"		*80*
Nine tens	" "	*ninety*,	"		*90*

When two figures are written together to express a number, the left-hand figure expresses *tens*, and the right-hand figure *ones*.

Sixteen	consists of	1	ten	and	6	ones,	written	*16*
Twenty-four	"	"	2	tens	"	4	"	"	*24*
Thirty-two	"	"	3	"	"	2	"	"	*32*
Forty-nine	"	"	4	"	"	9	"	"	*49*
Fifty-five	"	"	5	"	"	5	"	"	*55*
Sixty-seven	"	"	6	"	"	7	"	"	*67*
Seventy-three	"	"	7	"	"	3	"	"	*73*
Eighty	"	"	8	"	"	0	"	"	*80*
Ninety-one	"	"	9	"	"	1	one,	"	*91*

(See Manual, page 214.)

EXERCISES.

1. Write in words the following numbers: 14, 25, 37, 42, 56, 69, 71, 88, 93.

2. Express by figures the following numbers: twelve, twenty-eight, thirty-five, forty-one, fifty-nine, sixty-three, seventy-six.

3. Write in words, 17, 29, 30, 48, 52, 65, 70, 81, 99.

4. Express by figures, forty-three, sixty-six, ninety-five, fifteen, eighty-six, thirty-eight, fifty-seven, sixty-one.

5. Write in figures, twenty, forty, eleven, thirty-six, ninety-four, eighty-nine, forty-six, seventy-five.

6. Write in words, 13, 45, 51, 78, 83, 97.

7. Write in words, 60, 91, 79, 84, 27, 33.

8. Express by figures, twenty-two, thirty-nine, fifty-four, ninety-six, twenty-seven, sixty-two, fifty-three, seventy-four.

9. Write in figures, eighty-seven, ninety-two, thirty-four, twenty-six, seventy-two, sixty-eight, forty-four, ninety-eight.

10. Write in words, 19, 77, 51, 64, 23, 82, 58.

3. In writing numbers, every ten *tens* taken together are called a *hundred.*

One hundred is written				*100*
Twenty tens are	two hundred,	written		*200*
Thirty tens	" three hundred,	"		*300*
Forty tens	" four hundred,	"		*400*
Fifty tens	" five hundred,	"		*500*
Sixty tens	" six hundred,	"		*600*
Seventy tens	" seven hundred,	"		*700*
Eighty tens	" eight hundred,	"		*800*
Ninety tens	" nine hundred,	"		*900*

When three figures are written together to express a number, the left-hand figure expresses *hundreds*, the second or middle figure, *tens*, and the right-hand figure, *ones*. Thus, two hundred forty-three consists of 2 hundreds, 4 tens, and 3 ones, and is written 243.

The numbers in the first column below consist of hundreds, tens, and ones, as shown in the second column, and are written as in the third column.

One hundred forty-nine,	1	hundred,	4	tens,	and	9	ones,		*149*
Four hundred sixty-two,	4	hundreds,	6	"	"	2	"		*462*
Five hundred twenty,	5	"	2	"	"	0	"		*520*
Six hundred seventy,	6	"	7	"	"	0	"		*670*
Seven hundred five,	7	"	0	"	"	5	"		*705*
Eight hundred four,	8	"	0	"	"	4	"		*804*
Nine hundred twelve,	9	"	1	ten,	"	2	"		*912*

(See Manual, page 214.)

EXERCISES.

11. Write in words the numbers 247, 356, 528, 646, 935.

12. Express by figures, one hundred seventy-three, four hundred ninety-one, seven hundred sixty-four, and nine hundred eighty-two.

13. Write in words, 617, 121, 745, 514, 311.

14. Express by figures, four hundred nineteen, nine hundred thirty-nine, three hundred thirty-three, eight hundred eleven.

15. Write in words, 560, 310, 290, 420, 750.

16. Write in figures, one hundred thirty, six hundred forty, eight hundred eighty, four hundred ten.

17. Write in words, 208, 906, 301, 606, 807.

18. Express by figures, eight hundred two, one hundred nine, four hundred three, seven hundred five.

19. Write in words, 293, 780, 519, 103, 612, 999.

20. Express by figures the numbers which consist of 8 hundreds, 2 tens, and 4 ones; 2 hundreds, 1 ten, and 8 ones; 5 hundreds, 7 tens, and 0 ones; 9 hundreds, 5 tens, and 4 ones; 3 hundreds, 0 tens, and 7 ones.

4. In writing numbers, every ten hundreds taken together are called a *thousand.* Thousands are written thus:

One thousand, *1000*	Five thousand, *5000*
Two thousand, *2000*	Six thousand, *6000*
Three thousand, *3000*	Seven thousand, *7000*
Four thousand, *4000*	Eight thousand, *8000*

Nine thousand, *9000*

In any number written with more than three figures, the figure at the left of hundreds expresses thousands. Thus, 3579 consists of 3 thousands, 5 hundreds, 7 tens, and 9 ones; and expresses the number three thousand five hundred seventy-nine.

5. Every ten thousands taken together are called a *ten-thousand.*

6. Every ten ten-thousands taken together are called a *hundred-thousand.*

When a figure stands at the left of thousands, it expresses ten-thousands; and when a figure stands at the left of ten-thousands, it expresses hundred-thousands.

Ten thousand is written	*10000*
Two ten-thousands are twenty thousand, written	*20000*
Three ten-thousands are thirty thousand, "	*30000*
Eight ten-thousands are eighty thousand, "	*80000*
One hundred-thousand is written	*100000*
Two hundred thousands are written	*200000*
Five hundred thousands " "	*500000*

7. Every three figures in any number counting from the right are called a *Period.* Periods of figures are separated from each other by commas.

The first or right-hand period consists of ones, tens, and hundreds; and the second period of ones, tens, and hundreds of thousands. Thus,

of thousands.						
8	7	4,	2	3	5	
hundreds	tens	ones	hundreds	tens	ones	

Eighteen thousand five hundred thirty-six	is	written	*18,536*
Thirty-two thousand eight	"	"	*32,008*
Forty-seven thousand two hundred	"	"	*47,200*
Sixty thousand four hundred twenty	"	"	*60,420*
Two hundred forty thousand	"	"	*240,000*
Four hundred eight thousand five hundred	"	"	*408,500*
Five hundred thousand three hundred seventy-five	"	"	*500,375*
Six hundred fifty-two thousand ten	"	"	*652,010*
Seven hundred forty-four thousand six	"	"	*744,006*
Eight hundred fifty-three thousand five hundred seventy-six	"	"	*853,576*

(See Manual, page 214.)

EXERCISES.

21. Write in words, 8,000; 5,400; 2,560; 1,644; 3,729.

22. Write in words, 6,944; 3,405; 4,094; 7,010; 6,009.

23. Express by figures, five thousand three hundred fifty-six, seven thousand two hundred forty, one thousand nine hundred three, four thousand fifty-six.

24. Express by figures, nine thousand eight hundred, two thousand six, eight thousand fifty, five thousand, six thousand eight hundred nineteen, one thousand one.

25. Write in words, 15,380; 26,506; 37,081; 40,269; 93,274.

26. Write in words, 6,793; 72,400; 80,560; 63,004; 50,041.

27. Write in words, 33,000; 40,900; 90,209; 19,040; 20,007.

28. Express by figures, sixty thousand, fifty thousand twenty, nineteen thousand three hundred one, fifty-six thousand eleven.

29. Write in words, 132,041; 270,405; 320,500; 400,385.

30. Write in words, 574,000; 629,005; 700,044; 803,000; 957,503; 793,461; 809,051; 907,200.

31. Write in words, 503,020; 482,070; 600,002; 855,480.

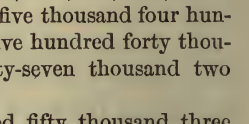
32. Write in words, 100,905; 350,240; 904,306; 100,040.

33. Express by figures, one hundred five thousand four hundred seventy, two hundred thousand, five hundred forty thousand seventy-two, seven hundred forty-seven thousand two hundred.

34. Express by figures, two hundred fifty thousand three hundred sixty-three, four hundred sixty thousand twenty, seven hundred ten thousand, eight hundred one thousand four.

35. Express by figures, two hundred thousand six hundred forty, three hundred five thousand two hundred ninety-four, six hundred eighty thousand five, nine hundred thousand six hundred.

8. The third period of figures consists of *ones*, *tens*, and *hundreds* of *millions*. Thus,



In any full period the right-hand figure is ones, the middle figure is tens, and the left-hand figure is hundreds. Thus, in any number consisting of three full periods, there are ones, ones of thousands, and ones of millions; tens, tens of thousands, and tens of millions; and hundreds, hundreds of thousands, and hundreds of millions.

Two million five hundred thousand eighty	is written		*2,500,080*
Thirty-four million three hundred twenty-four thousand five hundred eighty-six	"	"	*34,324,586*
Forty million forty-four thousand twelve	"	"	*40,044,012*
One hundred twenty-nine million three hundred seventeen thousand five hundred	"	"	*129,317,500*
Six hundred fifty million two hundred thousand seventy	"	"	*650,200,070*
Nine hundred three million fifty thousand five hundred ninety-four,	"	"	*903,050,594*
Three hundred million three thousand thirty	"	"	*300,003,030*

9. The writing of numbers in figures is *Notation.*

10. The reading of numbers which are expressed by figures is *Numeration.*

11. The place which any figure occupies in a number determines the value expressed by it in that number.

The values of the different places in their order are shown by the following

NOTATION AND NUMERATION TABLE.

10 ones	are 1 ten,
10 tens	" 1 hundred,
10 hundreds	" 1 thousand,
10 thousands	" 1 ten-thousand,
10 ten-thousands	" 1 hundred-thousand,
10 hundred-thousands	" 1 million,
10 millions	" 1 ten-million,
10 ten-millions	" 1 hundred-million.

1 ten	is 10 ones,
1 hundred	" 10 tens,
1 thousand	" 10 hundreds,
1 ten-thousand	" 10 thousands,
1 hundred-thousand	" 10 ten-thousands,
1 million	" 10 hundred-thousands,
1 ten-million	" 10 millions,
1 hundred-million	" 10 ten-millions.

(See Manual, page 214.)

REVIEW EXERCISES.

36. Write in words, 4,650; 738; 450,840; 93,066; 3,050,300; 1,005; 4,000,800.

37. Write in words, 37,098,420; 502,000; 730,900.

38. Write in words, 85,700,025; 6,000,030; 13,006,400; 45,000,000; 6,412; 2,578,024.

39. Write in words, 72,059,209; 10,765,291; 8,010; 70,045; 8,050,000.

40. Express by figures, seven hundred six thousand two hundred ninety-one, seventy-four thousand seven hundred four, nine thousand ninety, eighty thousand twelve.

41. Express by figures, eight million five thousand three hundred ninety-four, six million eight, seven million, three hundred thousand seven hundred twenty.

42. Express by figures, nine million two hundred seventy, two hundred three thousand four hundred five, twenty thousand six hundred seven, eighteen million nineteen.

43. Write in words, 4,080,306; 232,107,003; 14,200; 500,007; 60,572; 536,000.

44. Write in words, 34,000,709; 1,702,050; 605,400,300; 9,600,309.

45. Write in words, 12,065,587; 40,080,276; 7,200,000; 15,009,820; 3,031,504.

46. Express by figures, four million two hundred fifty-nine, seven million two hundred two thousand five hundred, eighty thousand, four hundred thousand two hundred fifty-six.

47. Express by figures, eight hundred million seven hundred seven thousand five hundred six, sixteen thousand sixteen, seven million five thousand forty-four, twenty-nine million forty-one thousand.

48. Express by figures, two hundred four thousand two hundred seventy, fifty thousand thirty-three, one million three hundred six, seven hundred fifty thousand nine.

49. Write in figures, ten million twenty-five thousand four hundred, six thousand one, three million two thousand, sixteen million ninety thousand five.

50. Write in words, 7,019,003; 506,427, 711; 242,424; 736,378; 9,999; 300,003,303.

SECTION II.

ADDITION.

INDUCTION.

(See Manual, page 215.)

12. HERE is a picture of some boys and girls in an orchard gathering apples. John has 1 apple in each hand; Harry has 2 apples in one hand and 1 in the other; Mary has 3 apples in one hand and 1 in the other; and Fanny has 4 apples in her lap and 1 in her hand.

1. How many apples has John? How many has Harry? How many has Mary? How many has Fanny?

2. How many apples have John and Harry together?

3. How many apples have Mary and Fanny?

4. If John and Harry give their apples to Mary, how many apples will she have?

5. If Harry and Mary give their apples to Fanny, how many will she have?

6. If all the children put their apples into Mary's basket, how many apples will be in the basket?

13. When two or more numbers are united to form one number, the process is *Addition.*

14. The result thus formed is the *Amount* or *Sum*, and the numbers to be united are the *Parts.*

The amount or sum must contain as many ones as all the parts taken together.

7. What is the amount of 11 cents, 5 cents, and 8 cents?

8. What is the amount of 8 pencils, 6 pencils, and 10 pencils?

9. What is the sum of 4 walnuts, 1 walnut, 2 walnuts, and 9 walnuts?

10 What is the sum of 2 days, 4 days, 3 days, and 6 days?

11. Add 8 peaches, 4 peaches, 3 peaches, and 7 peaches.

12. Add 5 roses, 12 roses, 7 roses, and 6 roses.

13. Martha has a 5-cent piece, a 3-cent piece, a 2-cent piece, and a 10-cent piece. What sum of money has she?

14. A laborer worked four weeks, earning 8 dollars the first week, 6 dollars the second, 4 dollars the third, and 7 dollars the fourth. What amount of money did he earn

15. Add 3 and 7 and 5 and 8 and 2.

16. Add 9 and 4 and 6 and 1 and 5.

15. This sign +, written between numbers, signifies that they are to be added.

It is called *Plus*, or the *Sign of Addition.*

16. This sign = written between numbers or sets of numbers, signifies that they are equal to each other.

It is called the *Sign of Equality.* Thus,

$6 + 7 + 12 = 25$ is read, 6 plus 7 plus 12 equal 25.

17. Read $7 + 3 + 5 = 15$.

18. Read $29 = 10 + 9 + 6 + 4$.

19. Read $12 + 5 + 4 = 13 + 8$.

20. 16 eggs + 5 eggs + 11 eggs + 9 eggs = how many eggs?

21. 8 hats + 21 hats + 6 hats + 10 hats = how many hats?

22. What is the sum of 7 chairs + 12 chairs + 6 chairs + 10 chairs + 1 chair?

23. What is the amount of 19 dollars + 10 dollars + 7 dollars + 6 dollars + 5 dollars?

24. 18 + 7 + 6 + 9 = how many? (See Manual, page 215.)

17. ADDITION TABLE.

0	0	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9	10
2	0	1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2	2	2
	2	3	4	5	6	7	8	9	10	11
3	0	1	2	3	4	5	6	7	8	9
	3	3	3	3	3	3	3	3	3	3
	3	4	5	6	7	8	9	10	11	12
4	0	1	2	3	4	5	6	7	8	9
	4	4	4	4	4	4	4	4	4	4
	4	5	6	7	8	9	10	11	12	13
5	0	1	2	3	4	5	6	7	8	9
	5	5	5	5	5	5	5	5	5	5
	5	6	7	8	9	10	11	12	13	14
6	0	1	2	3	4	5	6	7	8	9
	6	6	6	6	6	6	6	6	6	6
	6	7	8	9	10	11	12	13	14	15
7	0	1	2	3	4	5	6	7	8	9
	7	7	7	7	7	7	7	7	7	7
	7	8	9	10	11	12	13	14	15	16
8	0	1	2	3	4	5	6	7	8	9
	8	8	8	8	8	8	8	8	8	8
	8	9	10	11	12	13	14	15	16	17
9	0	1	2	3	4	5	6	7	8	9
	9	9	9	9	9	9	9	9	9	9
	9	10	11	12	13	14	15	16	17	18

ORAL EXERCISES.

1.—1. Count to 100 in this manner; 0 and 1 are 1, 1 and 1 are 2, 2 and 1 are 3, and so on.

2. Count to 100, thus; 0, 1, 2, 3, 4, 5, and so on.

2.—1. Count by 2's to 100, in this manner, 0 and 2 are 2, 2 and 2 are 4, 4 and 2 are 6, and so on.

2. Count by 2's to 100, thus; 0, 2, 4, 6, 8, and so on.

3. Commence with 1, and count by 2's to 101, thus; 1 and 2 are 3, 3 and 2 are 5, 5 and 2 are 7, and so on.

4. Count by 2's from 1 to 101, thus; 1, 3, 5, 7, 9, and so on.

3.—1. Count by 3's from 0 to 102, thus; 0 and 3 are 3, 3 and 3 are 6, 6 and 3 are 9, and so on.

2. Count by 3's from 0 to 102, thus; 0, 3, 6, 9, 12, and so on.

3. Commence with 1 and count by 3's to 100, thus; 1 and 3 are 4, 4 and 3 are 7, 7 and 3 are 10, and so on.

4. Count by 3's from 1 to 100, thus; 1, 4, 7, 10, 13, and so on.

5. Commence with 2 and count by 3's to 101, thus; 2 and 3 are 5, 5 and 3 are 8, 8 and 3 are 11, and so on.

6. Count by 3's from 2 to 101, thus; 2, 5, 8, 11, 14, and so on.

4.—1. Commence with 0 and count by 4's to 100, thus; 0 and 4 are 4, 4 and 4 are 8, 8 and 4 are 12, and so on.

2. Count by 4's from 0 to 100, thus; 0, 4, 8, 12, 16, and so on.

3. Commence with 1 and count by 4's to 101, thus; 1 and 4 are 5, 5 and 4 are 9, 9 and 4 are 13, and so on.

4. Count by 4's from 1 to 101, thus; 1, 5, 9, 13, 17, and so on.

5. Commence with 2 and count by 4's to 102, thus; 2 and 4 are 6, 6 and 4 are 10, 10 and 4 are 14, and so on.

6. Count by 4's from 2 to 102, thus; 2, 6, 10, 14, 18, and so on.

7. Commence with 3 and count by 4's to 103, thus; 3 and 4 are 7, 7 and 4 are 11, 11 and 4 are 15, and so on.

8. Count by 4's from 3 to 103, thus; 3, 7, 11, 15, 19, and so on.

5.—1. Commence with 0 and count by 5's to 100, thus; 0 and 5 are 5, 5 and 5 are 10, 10 and 5 are 15, and so on.

2. Count by 5's from 0 to 100, thus; 0, 5, 10, 15, and so on.

3. Commence with 1 and count by 5's to 101. (See Manual, page 215.)

4. Commence with 2 and count by 5's to 102.

5. Count by 5's from 3 to 103.

6. Count by 5's from 4 to 104.

6.—1. Count by 6's from 0 to 102, thus; 0 and 6 are 6, 6 and 6 are 12, 12 and 6 are 18, and so on.

2. Count by 6's from 0 to 102, thus; 0, 6, 12, 18, 24, and so on.

3. Commence with 1 and count by 6's to 103.

4. Count by 6's from 2 to 104.

5. Count by 6's from 3 to 105.

6. Count by 6's from 4 to 100.

7. Count by 6's from 5 to 101.

7.—1. Commence with 0 and count by 7's to 105, thus; 0 and 7 are 7, 7 and 7 are 14, 14 and 7 are 21, and so on.

2. Count by 7's from 0 to 105, thus; 0, 7, 14, 21, 28, and so on.

3. Commence with 1 and count by 7's to 106.

4. Commence with 2 and count by 7's to 100.

5. Commence with 3 and count by 7's to 101.

6. Count by 7's from 4 to 102.

7. Count by 7's from 5 to 103.

8. Count by 7's from 6 to 104.

8.—1. Commencing with 0, count by 8's to 104, thus; 0 and 8 are 8, 8 and 8 are 16, 16 and 8 are 24, and so on.

2. Count by 8's from 0 to 104, thus; 0, 8, 16, 24, 32, and so on.

3. Commencing with 1, count by 8's to 105.

4. Commencing with 2, count by 8's to 106.

5. Commencing with 3, count by 8's to 107.

6. Count by 8's from 4 to 100.

7. Count by 8's from 5 to 101.

8. Count by 8's from 6 to 102.

9. Count by 8's from 7 to 103.

9.—1. Commencing with 0, count by 9's to 108, thus; 0 and 9 are 9, 9 and 9 are 18, 18 and 9 are 27, and so on.
2. Count by 9's from 0 to 108, thus; 0, 9, 18, 27, 36, and so on.
3. Commencing with 1, count by 9's to 100.
4. Commencing with 2, count by 9's to 101.
5. Commencing with 3, count by 9's to 102.
6. Count by 9's from 4 to 103.
7. Count by 9's from 5 to 104.
8. Count by 9's from 6 to 105.
9. Count by 9's from 7 to 106.
10. Count by 9's from 8 to 107.

CASE I.

The sum of all the figures of any place not more than 9.

18. EXAMPLE. What is the sum of 2,344 and 3,152?

EXPLANATION.—Since these parts are too large to be added mentally, we write them one under the other, writing the *ones* of one part under the *ones* of the other, the *tens* under *tens*, the *hundreds* under *hundreds*, and the *thousands* under *thousands*. The sum of 2 *ones* and 4 *ones* is 6 *ones*, which we write under the *ones;* the sum of 5 *tens* and 4 *tens* is 9 *tens*, which we write under the *tens;* the sum of 1 *hundred* and 3 *hundreds* is 4 *hundreds*, which we write under the *hundreds;* and the sum of 3 *thousands* and 2 *thousands* is 5 *thousands*, which we write under the *thousands*. The result, 5,496, is the sum required.

SOLUTION.

2,344
3,152 } Parts.
5,496 Sum.

19. We can add apples to apples, dollars to dollars, pens to pens, or hours to hours; but we can not add apples to dollars, nor pens to hours. For 4 apples + 9 dollars = neither 13 apples nor 13 dollars.

Again, we can add ones to ones, tens to tens, or hundreds to hundreds; but we can not add ones to hundreds, nor tens to thousands. For 4 tens + 9 thousands = neither 13 tens nor 13 thousands. Hence,

20. *General Principles of Addition.*

I. *Only numbers expressing the same kind of things can be added.*

II. *Only figures occupying the same place in different numbers can be added;* that is, ones must be added to ones, tens to tens, hundreds to hundreds, thousands to thousands, and so on. (See Manual, page 215.)

PROBLEMS.

Find the sum of the numbers in each of the first ten problems.

(1)	(2)	(3)	(4)	(5)	(6)
62	26	34	452	281	504
24	72	145	37	612	283

(7)	(8)	(9)	(10)
235 men	2,413 books	5,241 miles	31,410 dollars
612 men	146 books	306 miles	1,245 dollars
141 men	30 books	2,432 miles	26,332 dollars

11. James paid 12 cents for a slate, and 15 cents for a writing-book. How many cents did he pay for both?

12. Myron found 25 plums under one tree in the garden, and 13 plums under another. How many plums did he find under both trees?

13. In a village school are 56 boys and 43 girls. How many pupils in the school?

14. One day a lady traveled 42 miles by railroad and 16 miles by stage. How many miles did she travel?

15. An orchard consists of 53 winter apple-trees and 14 fall apple-trees. How many trees are in the orchard?

16. A builder paid 610 dollars for a city lot, and built upon it a house which cost him 2,085 dollars. How much did the house and lot cost?

17. What is the sum of 542 + 36?

18. 21 + 45 + 32 = how many?

19. 6,132 + 31 + 36 = how many? *6,199.*

20. What is the sum of 122+231+312+123+201? *989.*

21. What is the sum of four hundred one thousand nine hundred fifty, twenty-four thousand twenty-four, and two thousand and four? *427,978.*

22. A farmer harvests from five fields of wheat, 151 bushels, 204 bushels, 120 bushels, 312 bushels, and 211 bushels. How many bushels of wheat did he harvest? *998.*

23. In January a laborer deposited in the savings bank 12 dollars, in February 30 dollars, in March 13 dollars, in April 11 dollars, in May 21 dollars, and in June 12 dollars. How many dollars did he deposit in the six months? *99 dollars.*

24. One week in May one dairyman furnished to a cheese factory 2,432 pounds of milk, another dairyman 4,145 pounds, and another 3,221 pounds. How many pounds were furnished by the three dairymen? *9,798.*

25. The amount of cheese manufactured at the same factory in June was 12,147 pounds, in July 13,410 pounds, in August 22,221 pounds, and in September 11,211 pounds. How many pounds were manufactured in the four months? *58,989.*

26. At a cotton factory 1,465,207 yards of cloth were made in 1864, and 1,532,492 yards in 1865. How many yards were made in the two years? *2,997,699.*

27. A grocer bought four hogsheads of sugar, weighing 1,154 pounds, 1,213 pounds, 1,301 pounds, and 1,231 pounds. How many pounds did they all weigh? *4,899.*

28. Long Island consists of three counties. Kings County contains 72 square miles, Queens County 410 square miles, and Suffolk County 1,200 square miles. How many square miles in the island? *1,682.*

29. A fruit-grower sold 123 barrels of apples to one man, 31 barrels to another, 103 to a third, 30 barrels to a fourth, and 112 barrels to a fifth. How many barrels of apples did he sell? *399.*

30. The mason work of a new school-house cost 1,220 dollars, the carpenter work 1018 dollars, and the painting and glazing 430 dollars. How many dollars did the school-house cost? *2,668.*

CASE II.

The sum of all the figures of any place more than 9.

FIRST METHOD.

21. Ex. What is the sum of 28, 76 and 39?

EXPLANATION. — 1*st. Writing the numbers.* — We write ones under ones, and tens under tens, and below the last number we draw two parallel horizontal lines, far enough apart to allow us to write figures between them.

FIRST STEP.

28
76
39
—
—

2*d. Adding the Numbers.*—Adding the ones, we find the sum to be 23, or 3 ones and 2 tens. We write the 3 ones below the lower line as the ones of the required sum, and, since we must add the 2 tens to the tens of the given numbers, we write them in tens' place, between the two lines. Adding the tens, we find the sum to be 14, or 4 tens and 1 hundred. As there are no hundreds in the given numbers to which to add this 1 hundred, we write the 4 tens and the 1 hundred below the lower line, as tens and hundreds of the required sum. The result, 143, is the sum required. (See Manual, page 215.)

SECOND STEP.

28
76
39
—
2
—
3

SOLUTION.

28 } Parts.
76 }
39 }
—
2
—
143

PROBLEMS.

31. A farmer has 46 sheep in one flock and 38 in another. How many sheep has he? *84.*

32. A merchant sold 13 yards of calico to one lady, 14 yards to another, and 16 yards to another. How many yards did he sell to the three ladies? *43.*

33. Two wood-choppers worked together through the winter. One of them chopped 174 cords of wood and the other 167 cords. How many cords did both of them chop? *341.*

34. On this map of a farm, how many acres of woodland on both sides of Willow Pond? (See Manual p. 215.)

35. How many acres of pasture on both sides of Stony Brook?

36. How many acres are in the two meadows?

37. How many acres of tilled land does the farm contain? *71.*

38. How many acres are on the east side of Willow Pond and Stony Brook?

39. How many acres on the west side?

40. How many acres in the farm including Willow Pond? *256.*

41. The orchard is 47 rods long and 34 rods wide. How many rods long is the stone fence around it? *162.*

42. The meadow north of the orchard is 66 rods long and 58 rods wide. How many rods of fence around it?

43. The whole pasture is 110 rods long and 64 rods wide. How many rods of rail fence on the three sides, as shown on the map? *284.*

44. The yard and garden are 34 rods long and 19 rods wide. In front is a picket fence, and on the other three sides is stone fence. How many rods long is the stone fence?

45. How many rods in the fences which inclose the yard and garden?

46. The lengths of the different fences inclosing the farm are shown on the map. How many rods of these are stone fence? *237.*

47. How many rods are rail fence?

48. How many rods of fence of all kinds around the farm?

49. How many rods of road in front of this farm?

Table of Railroad Distances between Boston and St. Louis.

Principal Stations.	Distances in miles.	Principal Stations.	Distances in miles.	Principal Stations.	Distances in miles.
Boston, . . .		Buffalo, . . .		Chicago, . . .	
Worcester, . .	44	Dunkirk, . .	40	Joliet, . . .	36
Springfield, .	54	Erie, ...	48	Bloomington, .	88
Pittsfield, . .	53	Cleveland, . .	95	Springfield, . .	60
Albany, . . .	49	Sandusky City,	61	Alton, . . .	72
Albany, . . .		Toledo, . . .	51	St. Louis, . .	25
Schenectady, .	17	Toledo, . . .		(See Manual, page 215.)	
Utica, . . .	78	Adrian, . . .	32		
Syracuse, . .	52	Coldwater, . .	56		
Rochester, . .	82	South Bend, .	69		
Batavia, . . .	32	La Porte, . .	27		
Buffalo, . . .	36	Chicago, . . .	59		

50. How many miles from Boston to Albany? *200.*

51. How many miles from Albany to Buffalo? *297.*

52. What is the distance from Buffalo to Toledo?

53. What is the distance frem Toledo to Chicago?

54. How far is it from Chicago to St. Louis? *281 miles.*

55. How far is it from Albany to Chicago? *835 miles.*

56. What is the distance from Boston to St. Louis?

57. One day a miller bought 1,284 bushels of wheat, and the next day 859 bushels. How many bushels did he buy in the two days? *2,143.*

58. A butcher killed an ox, the quarters of which weighed respectively 136 pounds, 143 pounds, 178 pounds, and 187 pounds. What was the weight of the four quarters? *644.*

59. A grocer bought five jars of butter, containing respectively 33 pounds, 47 pounds, 32 pounds, 54 pounds, and 45 pounds. How many pounds of butter did the five jars contain?

60. One month a woolen manufacturer paid out 31,587 dollars for stock, and 23,476 dollars for labor. How many dollars did he pay out during the month? *55,063.*

61. 53 feet + 171 feet + 23,869 feet + 24 feet + 359,487 feet = how many feet? *383,604 feet.*

SECOND METHOD.

22. Ex. Add 346, 5,279, and 8,165.

SOLUTION.

346
5,279
8,165
13,790

EXPLANATION.—After writing the parts, with ones under ones, tens under tens, and so on, we draw one horizontal line under the last number. Adding the ones, we find the sum to be 20, or 0 ones and 2 tens. We write the 0 ones below the line in the ones' place of the required sum; and the 2 tens we add with the tens of the given numbers, but without first writing it in a line by itself. The sum of all the tens is 19, or 9 tens and 1 hundred. We write the 9 tens below the line as the tens of the required sum; and the 1 hundred we add with the hundreds of the given numbers. The sum of all the hundreds is 7, which we write below the line in hundreds' place. The sum of all the thousands is 13, which we write below the line as the thousands and ten-thousands of the required sum. The result, 13,790, is the sum required. (See Manual, page 216.)

PROBLEMS.

62. In the first passenger car of a railroad train were 49 passengers, in the second 63, in the third 54, in the fourth 62, and in the fifth 48. How many passengers were on the train?

63. A railroad company purchased in one day 167 cords of wood at one station, 289 cords at another, 84 cords at another, and 417 cords at another. How many cords were purchased at the four stations? *957 cords.*

64. A merchant by selling a lot of damaged goods for $587, lost $94. How much did the goods cost him? *$681.*

A number with the sign $ before it expresses dollars.

65. Three men engaged in business together, the first furnishing $3,425 dollars, the second $2,163 dollars, and the third $896. What was the amount of their capital? *$6,484.*

66. A grain-buyer in Chicago paid $7,594 for a cargo of wheat, shipped it to New York at an expense of $2,841, and sold it so as to gain $1,565. For how much did he sell it?

67. A merchant pays for rent of store 1,275 dollars a year, for clerk-hire 3,895 dollars, for fuel 242 dollars, for gas 437 dollars, for freight and cartage on goods 936 dollars, and for other expenses 359 dollars. What is the amount of his yearly expenses? *$7,144.*

68. At a sale of government vessels, August 10, 1865, the bark Restless was sold for $12,000, the tug Larkspur for $8,100, the side-wheel steamer Alabama for $28,000, the schooner Matthew Vassar for $7,300, and the steam packet-boat Hartford for $9,100. For how much were all these vessels sold? *$64,500.*

69. A store in a brick building rents for $365 a year, the offices in the second story rent for $162, and a daguerrean room in the third story rents for $78. How much is the whole rent of the building? *$605.*

70. A merchant's cash sales on Monday were $96, Tuesday $132, Wednesday $98, Thursday $72, Friday $115, and Saturday $149. What was the amount of his cash sales for the week? *$662.*

71. One season a farmer killed six hogs which weighed 427 pounds, 329 pounds, 314 pounds, 217 pounds, 208 pounds, and 196 pounds. How much did they all weigh? *1,691 pounds.*

72. Seven rafts of saw-logs from Alleghany River passed Pittsburg in one day. The first raft contained 276 logs, the second 359, the third 409, the fourth 293, the fifth 318, the sixth 325, and the seventh 358. How many logs in all the rafts? *2,338 logs.*

(73)	(74)	(75)	(76)	(77)
30,076	141	28	14,193	647,129,341
5,821	30,648	52	6,009	327,293
498	8,291	164	417	284,384
167	287	386	1,306	43,100,085
22,849	165	1,227	129	2,873
3,482	24	2,873	873	541
691	2,841	642	154,685	30,698
482	596	578	7,676	28,165
1,642	417	249	48	475
56	13,509	3,871	509	1,465,127

23. *Rule for Addition of Integers.*

I. *Write the numbers to be added with ones under ones, tens under tens, hundreds under hundreds, and so on.*

II. *Add the column of ones, and, if the sum does not exceed 9, place it under the ones; but if it exceeds 9, place the right-hand figure under the ones.*

III. *Add the column of tens, and with it the left-hand figure of the sum of the ones, and if the sum does not exceed 9, place it under the tens; but if it exceeds 9, place the right-hand figure under the tens.*

IV. *Proceed in the same manner with each column successively, and write down the whole sum of the left-hand column.*

PROBLEMS.

78. A builder received $17,525 for erecting a church, $2,485 for building a dwelling; $580 for building a barn, and $265 for repairs on a store. How much did he receive for the four jobs? *$20,855.*

79. England contains 57,101 square miles, Scotland 31,324 square miles, Ireland 32,512 square miles, Wales 7,219 square miles, and the smaller British islands contain 324 square miles. How many square miles in the whole of Great Britain?

80. What is the sum of thirty-five million eight hundred seventy-six thousand one hundred twenty, three hundred ninety-six thousand four hundred ninety-one, and five hundred forty-three thousand six hundred seven? *36,816,218.*

81. One year a farmer raised 587 bushels of wheat, 1,229 bushels of oats, 643 bushels of corn, 184 bushels of rye, 259 bushels of barley, and 296 bushels of buckwheat. How many bushels of grain did he raise? *3,198 bushels.*

82. A man paid $3,478 for a farm, $1,117 for live stock, $635 for farming implements, $423 for grain and seeds, and $189 for repairing fences and buildings. How much was his total outlay? *$5,842.*

83. A pork-packer in Cincinnati packed 15,287 barrels of pork in December, 13,164 barrels in January, and 9,645 barrels in February. How many barrels did he pack in the three months? *38,096 barrels.*

84. One day five fishing-smacks entered the harbor of Marblehead, bringing respectively 147 barrels of mackerel, 204 barrels, 89 barrels, 246 barrels, and 94 barrels. How many barrels of mackerel did all of them bring? *780 barrels.*

85. A drover paid $5,897 for 465 head of cattle, $3,486 for 284 head, $9,784 for 587 head, and $2,563 for 108 head. How many cattle did he buy, and how much did he pay for them? *1,444 head of cattle; $21,730.*

86. A merchant buys a bale of sheeting, containing 3 pieces of 38 yards each, 4 pieces of 39 yards each, 6 pieces of 42 yards each, and 5 pieces of 40 yards each. How many pieces in the bale? How many yards? *722 yards.*

87. At the battle of Gettysburg the loss in the Union army was 2,834 men killed and 13,790 wounded, and in the Confederate army 4,500 killed and 26,500 wounded. What was the whole loss in each army? *Union, 16,624; Confederate, 31,000.*

88. What was the whole number of men killed? *7,334.*

89. What was the whole number wounded? *40,290.*

90. What was the whole loss in both armies? *47,624.*

91. The number of cattle received at the New York Cattle Market in one week was 226 by the New York and Erie Railroad, 116 by the Hudson River Railroad, 2,669 by the Harlem Railroad, 319 by the New Jersey Central Railroad, 445 by Hudson River boats, and 26 on foot. How many cattle were received that week? *3,801.*

92. The value of the gold and silver exported from California in ten years commencing with 1854, was as follows:

In 1854, . .	$52,045,633	In 1859, . .	$47,640,462
In 1855, . .	45,161,731	In 1860, . .	42,325,916
In 1856, . .	50,697,434	In 1861, . .	40,676,758
In 1857, . .	48,976,697	In 1862, . .	42,561,761
In 1858, . .	47,548,026	In 1863, . .	46,071,920

What was the total value for the ten years? *$463,706,338.*

SECTION III.

SUBTRACTION.

INDUCTION.

(See Manual, page 216.)

24. 1. Of the 8 ladies in this picture, 3 are coming down the street, and the others are going up the street. How many ladies are going up the street?

2. Four of the ladies are walking, and the others are riding. How many are riding?

3. In the picture are 9 horses, going up street, and the others coming down. How many horses are coming down street?

4. All but 3 of the 9 horses are driven in teams. How many are driven in teams?

5. There are 12 barrels in the picture, 5 of them on a cart, and the others by the store on the corner. How many are by the store?

6. Of the 13 men shown in the picture, 6 are walking, and the others are riding. How many are riding?

7. Of the 13 men, 10 are coming toward us, and the others are going from us. How many are going from us?

25. When one of two numbers is taken from the other, the process is *Subtraction.*

26. The result thus found is the *Remainder,* or *Difference.*

27. The number from which another is to be taken is the *Minuend.*

28. The number to be taken from another is the *Subtrahend.*

The number of ones in the subtrahend and remainder, taken together, must equal the number of ones in the minuend.

8. Subtract 7 books from 11 books.

9. What will be the remainder if you take 9 chairs from 16 chairs?

10. If 6 cents be subtracted from 15 cents, what will be the remainder?

11. A cook, having 18 eggs, used 9 on Monday, and the remainder on Tuesday. How many did she use on Tuesday?

12. What is the difference between 17 leaves and 8 leaves?

13. How much is the difference between 14 bushels of potatoes and 5 bushels of potatoes?

14. From 15 inches subtract 7 inches.

15. Subtract 9 from 19.

16. The minuend is 13, and the subtrahend 4. What is the remainder?

29. This sign —, written between two numbers, signifies that the number after it is to be subtracted from the number before it.

It is called *Minus,* or the *Sign of Subtraction.* Thus, $25 - 16 = 9$ is read 25 minus 16 equals 9.

17. Read $15 - 7 = 8$.

18. Read $17 - 6 = 11$.

19. Read $13 + 8 = 30 - 9$.

20. $18 - 7 =$ how many?

21. 21 brushes — 11 brushes = how many brushes?

22. 27 words — 8 words — 5 words — 7 words = how many words?

(See Manual, page 216.)

30. SUBTRACTION TABLE.

0	0	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9
2	2	3	4	5	6	7	8	9	10	11
	2	2	2	2	2	2	2	2	2	2
	0	1	2	3	4	5	6	7	8	9
3	3	4	5	6	7	8	9	10	11	12
	3	3	3	3	3	3	3	3	3	3
	0	1	2	3	4	5	6	7	8	9
4	4	5	6	7	8	9	10	11	12	13
	4	4	4	4	4	4	4	4	4	4
	0	1	2	3	4	5	6	7	8	9
5	5	6	7	8	9	10	11	12	13	14
	5	5	5	5	5	5	5	5	5	5
	0	1	2	3	4	5	6	7	8	9
6	6	7	8	9	10	11	12	13	14	15
	6	6	6	6	6	6	6	6	6	6
	0	1	2	3	4	5	6	7	8	9
7	7	8	9	10	11	12	13	14	15	16
	7	7	7	7	7	7	7	7	7	7
	0	1	2	3	4	5	6	7	8	9
8	8	9	10	11	12	13	14	15	16	17
	8	8	8	8	8	8	8	8	8	8
	0	1	2	3	4	5	6	7	8	9
9	9	10	11	12	13	14	15	16	17	18
	9	9	9	9	9	9	9	9	9	9
	0	1	2	3	4	5	6	7	8	9

ORAL EXERCISES.

1.—1. Subtract 1 from every number from 100 down to 1, thus; 1 from 100 leaves 99, 1 from 99 leaves 98, 1 from 98 leaves 97, and so on.

2. Count from 100 down to 1, thus; 100, 99, 98, 97, and so on.

2.—1. Subtract 2 from every second number from 100 down to 0, thus; 2 from 100 leaves 98, 2 from 98 leaves 96, 2 from 96 leaves 94, and so on.

2. Count by 2's from 100 down to 0, thus; 100, 98, 96, 94, and so on.

3. Subtract 2 from every second number from 101 down to 1, thus; 2 from 101 leaves 99, 2 from 99 leaves 97, 2 from 97 leaves 95, and so on.

4. Count by 2's from 101 down to 1, thus; 101, 99, 97, 95, and so on.

3.—1. Subtract 3 from every third number from 100 down to 1, thus; 3 from 100 leaves 97, 3 from 97 leaves 94, and so on.

2. Count by 3's from 100 down to 1, thus; 100, 97, 94, 91, and so on.

3. Count by 3's from 101 down to 2. (See Manual, page 216)

4. Count by 3's from 102 down to 0.

4.—1. Subtract 4 from every fourth number from 100 down to 0, thus; 4 from 100 leaves 96, 4 from 96 leaves 92, 4 from 92 leaves 88, and so on.

2. Count by 4's from 100 down to 0, thus; 100, 96, 92, 88, 84, and so on.

3. Count by 4's from 101 down to 1.

4. Count by 4's from 102 down to 2.

5. Count by 4's from 103 down to 3.

5.—1. Subtract 5 from every fifth number from 100 down to 0, thus; 5 from 100 leaves 95, 5 from 95 leaves 90, and so on.
2. Count by 5's from 100 down to 0, thus; 100, 95, 90, 85, and so on.
3. Count by 5's from 101 down to 1.
4. Count by 5's from 102 down to 2.
5. Count by 5's from 103 down to 3.
6. Count by 5's from 104 down to 4.

6.—1. Subtract 6 from every sixth number from 102 down to 0, thus; 6 from 102 leaves 96, 6 from 96 leaves 90, and so on.
2. Count by 6's from 102 down to 0, thus; 102, 96, 90, 84, and so on.
3. Count by 6's from 103 down to 1.
4. Count by 6's from 104 down to 2.
5. Count by 6's from 105 down to 3.
6. Subtract 6 from every sixth number from 100 down to 4.
7. Count by 6's from 101 down to 5.

7.—1. Subtract 7 from every seventh number from 105 down to 0, thus; 7 from 105 leaves 98, 7 from 98 leaves 91, and so on.
2. Count by 7's from 105 down to 0, thus; 105, 98, 91, 84, and so on.
3. Count by 7's from 106 down to 1.
4. Count by 7's from 100 down to 2.
5. Subtract 7 from every seventh number from 101 down to 3.
6. Subtract 7 from every seventh number from 102 down to 4.
7. Count by 7's from 103 down to 5.
8. Count by 7's from 104 down to 6.

8.—1. Subtract 8 from every eighth number from 104 down to 0, thus; 8 from 104 leaves 96, 8 from 96 leaves 88, and so on.
2. Count by 8's from 104 down to 0, thus; 104, 96, 88, 80, and so on.
3. Subtract 8 from every eighth number from 105 down to 1.
4. Count by 8's from 106 down to 2.
5. Count by 8's from 107 down to 3.
6. Count by 8's from 100 down to 4.
7. Subtract 8 from every eighth number from 101 down to 5.
8. Count by 8's from 102 down to 6.
9. Count by 8's from 103 down to 7.

9.—1. Subtract 9 from every ninth number from 108 down to 0, thus; 9 from 108 leaves 99, 9 from 99 leaves 90, and so on.
2. Count by 9's from 108 down to 0, thus; 108, 99, 90, 81, and so on.
3. Subtract 9 from every ninth number from 100 down to 1.
4. Count by 9's from 101 down to 2.
5. Count by 9's from 102 down to 3.
6. Subtract 9 from every ninth number from 103 down to 4.
7. Subtract 9 from every ninth number from 104 down to 5.
8. Count by 9's from 105 down to 6.
9. Count by 9's from 106 down to 7.
10. Count by 9's from 107 down to 8.

CASE I.

No figure of the subtrahend greater than the corresponding figure of the minuend.

31. Ex. What is the difference between 8,397 and 3,265?

EXPLANATION.—Since these numbers are too large to be subtracted mentally, we write the subtrahend below the minuend, with the *ones* under *ones*, the *tens* under *tens*, the *hundreds* under *hundreds*, and the *thousands* under *thousands*. Commencing at the right, we take the 5 *ones* from the 7 *ones*, and the remainder, 2 *ones*, we write under the *ones*. We next take the 6 *tens* from the 9 *tens*, and the remainder, 3 *tens*, we write under the *tens*. Then 2 *hundreds* from 3 *hundreds* leave 1 *hundred*, which we write under the *hundreds*; and 3 *thousands* from 8 *thousands* leave 5 *thousands*, which we write under the *thousands*. The result, 5,132, is the difference or remainder required.

SOLUTION.

8,397 Minuend.
3,265 Subtrahend.
5,132 Difference.

32. We can subtract apples from apples, dollars from dollars, pens from pens, or hours from hours; but we can not subtract apples from dollars, nor pens from hours. For 13 apples − 4 dollars = neither 9 apples nor 9 dollars.

Again, we can subtract ones from ones, tens from tens, or hundreds from hundreds; but we can not subtract ones from hundreds, nor tens from thousands. For 9 thousands − 4 tens = neither 5 tens nor 5 thousands. Hence,

33. *General Principles of Subtraction.*

I. *Only numbers expressing the same kind of things can be subtracted the one from the other.*

II. *Only figures occupying the same place in different numbers can be subtracted the one from the other.*

(See Manual, page 216.)

PROBLEMS.

(1)	(2)	(3)	(4)	(5)	(6)
62	76	45	57	682	584
41	24	34	43	350	302

(7)	(8)	(9)	(10)
635 pins	3,846 soldiers	7,968 shingles	57,908 pounds
412 pins	2,534 soldiers	5,453 shingles	43,700 pounds

11. James, having 27 marbles, gave 12 of them to John. How many marbles had he left? *15.*

12. From a piece of muslin containing 39 yards, a merchant sold 13 yards for a dress. How many yards remained in the piece? *26.*

13. Joseph had 46 cents, but he has spent 25 cents for a knife. How many cents has he now?

14. Ellen attended school 63 days in a term of 75 school-days. How many days was she absent? *12.*

15. A gardener picked 68 boxes of strawberries one forenoon, and 54 boxes in the afternoon. How many more boxes did he pick in the forenoon than in the afternoon?

16. Hiram lives 98 rods from the schoolhouse, and Thomas 41 rods. How much farther does Hiram walk in going to school than Thomas? *57 rods.*

17. 435 miles — 314 miles = how many miles? *121.*

18. 6,798 bushels — 2,641 bushels = how many bushels?

19. How many tons are 38,156 tons — 14,044 tons? *24,112.*

20. A fruit-dealer, having 247 baskets of peaches, sold 125 baskets. How many baskets had he left? *122.*

21. A man whose income is $875 a year, expends $734. How much money does he save? *$141.*

22. A drover bought a lot of cattle for $4,574, and sold them for $5,896. How much did he gain? *$1,322.*

(23)	(24)	(25)	(26)
57,698	675,004	2,174,943	167,065,149
43,257	245,002	42,301	4,042,136

27. One year a farmer raised 1,898 bushels of oats, and sold 1,427 bushels. How many bushels did he keep for use? *471.*

CASE II.

Any figure of the subtrahend greater than the corresponding figure of the minuend.

FIRST METHOD.

34. Ex. 1. From 16 subtract 9.

(See Manual, page 216.)

EXPLANATION.—We write the numbers as in Case I; but as we can not subtract 9 ones from 6 ones, we must unite the 1 ten, which equals 10 ones, with the 6 ones, and subtract the 9 from the whole 16 at once.

SOLUTION.

$$\begin{array}{r} 16 \\ 9 \\ \hline 7 \end{array}$$

Ex. 2. From 76 subtract 29.

EXPLANATION.—As we can not subtract 9 ones from 6 ones, we take 1 of the 7 tens and unite it with the 6 ones, making 16 ones, and subtracting 9 ones from the 16 ones, we write the remainder, 7, as the ones of the final result. Since we have already used one of the 7 tens, we have now only 6 tens in the minuend, and hence we subtract the two tens from 6 tens, and write the remainder, 4 tens, as the tens of the final result.

SOLUTION.

$$\begin{array}{r} {\scriptstyle 6\ 16} \\ 7\,\not{6} \\ 2\,9 \\ \hline 4\,7 \end{array}$$

PROBLEMS.

28. In a public school are 45 pupils, and 28 of them are girls. How many are boys? *17.*

29. A grocer sold 35 bars of soap from a box that contained 64 bars. How many bars were left in the box? *29.*

30. One day 92 boats passed Lockport on the Erie Canal, and 47 of them were going east. How many were going west?

31. A washerwoman had 72 clothes-pins, but she has lost 29 of them. How many has she now? *43.*

32. A jar filled with butter weighs 52 pounds, and the jar alone weighs 15 pounds. How much does the butter weigh?

33. From a barrel of sugar containing 283 pounds, a merchant sold 156 pounds. How many pounds were left in the barrel? *127.*

34. A man bought a village lot for $350, and paid down all but $125. How much did he pay?

35. A man bought a piano for $475. He paid $267 in cash, and gave his note for the balance. For what sum did he give his note? *$208.*

35. Ex. From 853 subtract 467.

SOLUTION.

$$\begin{array}{r} {\scriptstyle 7\ 14\ 13} \\ 8\,5\,3 \\ 4\,6\,7 \\ \hline 3\,8\,6 \end{array}$$

EXPLANATION.—As we can not subtract 7 ones from 3 ones, we take 1 of the 5 tens and unite it with the 3 ones, making 13 ones; and subtracting 7 ones from 13 ones, we write the remainder, 6, as the ones of the final result. Since we have already used 1 of the 5 tens, only 4 tens now remain in the minuend. As we can not subtract the 6 tens from 4 tens, we unite 1 of the 8 hundreds with the 4 tens, making 14 tens; then subtracting 6 tens from 14 tens, we write the remainder, 8 tens, as the tens of the final result. Since we have already used 1 of the 8 hundreds, only 7 hundreds now remain in the minuend; and from this we subtract the 4 hundreds, and write the remainder, 3 hundreds, as the hundreds of the final result.

PROBLEMS.

36. A man who had a farm of 154 acres, gave to his son 65 acres. How much land had he left? *89 acres.*

37. In a certain village school-district are 447 children, of whom only 298 attend school. How many do not attend school? *149 children.*

38. A provision dealer receiving an order for 525 barrels of beef, has only 354 barrels on hand. How many barrels more will he require to fill the order?

39. A merchant's sales in January amounted to $1743, and in February to $928. How much did the sales of January exceed those of February? *$815.*

40. In a certain town 135 men were drafted for the army, but 54 of them were rejected by the examining surgeon. How many passed examination?

41. A regiment entered the service with 1,149 men, and at the close of the war had only 427. How many men had it lost? *722.*

42. A banker's income last year was $12,849, and his expenses were $6,768. How much did his income exceed his expenses? *$6,081.*

43. A market-gardener in one year received $3,730 for fruits and vegetables, and his expenses were $1,850. How much were his profits? *$1,880.*

44. One day 724 cattle were received at the Philadelphia Cattle Market, and 648 of them were sold. How many remained unsold?

45. A man having $974 in the bank, drew out $396. How much money had he left on deposit? *$578.*

36. Ex. From 3000 subtract 57.

SOLUTION.

$$\begin{array}{r} {\scriptstyle 2\ 9\ 9\ 10} \\ 3000 \\ 57 \\ \hline 2943 \end{array}$$

EXPLANATION.—We can not subtract 7 ones from 0 ones, and as we have in the minuend 0 tens to unite with the ones, and 0 hundreds to unite with the tens, we must take 1 of the 3 thousands, leaving 2 thousands. This 1 thousand = 10 hundreds; but as we can not subtract ones from hundreds (Prin. II.), we take 1 of the 10 hundreds, leaving 9 hundreds. This 1 hundred = 10 tens; but as we can not subtract ones from tens, we take 1 of the 10 tens, leaving 9 tens. This 1 ten = 10 ones. We now take the 7 ones from 10 ones, and the remainder, 3 ones, we write as the ones of the final result. Then 5 tens from 9 tens leave 4 tens, and as there are no hundreds or thousands in the subtrahend, we write the 9 hundreds and the 2 thousands of the minuend, for the hundreds and thousands of the final result.

PROBLEMS.

46. If I buy a bushel of apples for 65 cents, and give in payment a dollar bill, how much change should I receive?

47. At a flouring-mill in Baltimore 1000 barrels of flour were made in one week, and 869 barrels of it were sold. How many barrels were unsold? *131.*

48. The Phillips Well on Oil Creek is 460 feet deep, and the Titusville Well 1100 feet deep. How much deeper is the latter well than the former? *640 feet.*

49. A man divided $7,500 between his son and daughter, giving $4,275 to his son. How much did the daughter receive? *$3,225.*

50. A shipbuilder received $21,000 for a schooner, which cost him $18,728. How much was his gain? *$2,272.*

51. A forwarder had 40,000 bushels of oats in store at Chicago. How many bushels had he in store after shipping 25,487 bushels to Buffalo? *14,513 bushels.*

52. A broker sold stocks for $256,200 which cost him $209,408. Did he gain or lose, and how much? *Gained $46,792.*

53. How many acres are 1,100 acres — 841 acres? *259.*

54. How many cords are 21,610 cords — 19,587 cords?

55. How many gallons are 110,040 gallons—90,621 gallons?

56. At the battle of Bunker Hill the Americans lost 449 men, and the British 1054. How much did the British loss exceed the American?

SECOND METHOD.

37. Ex. From 7,623 subtract 4,856.

SOLUTION.

$$\begin{array}{r} 7,623 \\ 4,856 \\ \hline 2,767 \end{array}$$

EXPLANATION. — In this solution we commence at the right, and proceed the same as in the First Method, except that we omit to write the partial minuends, 13 ones, 11 tens, 15 hundreds, and 6 thousands, above the given minuend.

(See Manual, page 216.)

38. *Rule for Subtraction of Integers.*

I. *Write the subtrahend below the minuend, placing ones under ones, tens under tens, and so on.*

II. When the figures of the subtrahend do not exceed in value the corresponding figures of the minuend,

1. *Commencing at the right hand, subtract each figure of the subtrahend from the corresponding figure of the minuend, and write the result directly below in the remainder.*

2. *If there are figures in the minuend without any corresponding figures in the subtrahend, write them in the remainder.*

III. When any figure of the subtrahend exceeds the corresponding figure of the minuend,

Add 10 *to the figure of the minuend, and from the sum subtract the figure of the subtrahend. In this case, always call the next left-hand figure of the minuend* 1 *less, or the next left-hand figure of the subtrahend* 1 *more.*

(See Manual, page 216.)

PROBLEMS.

57. A load of hay with the wagon weighed 2,656 pounds on the hay scales, and the wagon alone weighed 987 pounds. How much did the hay weigh? *1,669 pounds.*

58. At a certain election 2,649 votes were cast for one candidate, and 1,975 votes for the other. What majority did the successful candidate receive? *674 votes.*

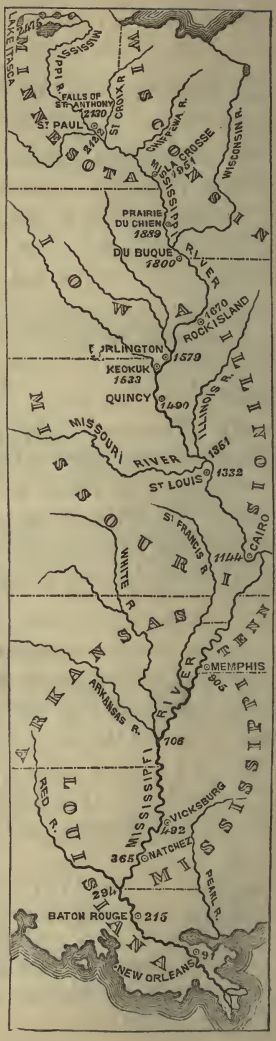
(59)	(60)	(61)	(62)
1,000,000	348,794	7,408,215	300,300,333
31,276	127,586	59,326	47,008,296

63. The island of Cuba contains 45,277 square miles, and the State of Ohio 39,904 square miles. How much larger is Cuba than Ohio?

The numbers on this map of Mississippi River show the distances of the different places from the mouth of the river.

How many miles is it from

64. Rock Island to Vicksburg?
65. Memphis to La Crosse? *1,046.*
66. Cairo to Lake Itasca? *1,331.*
77. St. Paul to Baton Rouge?
68. The Falls of St. Anthony to the mouth of Red River? *1,836.*
69. Burlington to Natchez? *1,214.*
70. The mouth of Missouri River to Prairie Du Chien? *538.*
71. New Orleans to Quincy? *1,399.*
72. Quincy to St. Paul? *632.*
73. The mouth of Arkansas River to the mouth of Missouri River? *645.*
74. Keokuk to New Orleans?
95. Du Buque to Lake Itasca?
76. Prairie Du Chien to Natchez?
77. St. Louis to the Falls of St. Anthony? *798.*
78. Memphis to Baton Rouge?
79. Keokuk to La Crosse? *418.*
80. Du Buque to Vicksburg?
81. Cairo to the mouth of Red River? *850.*
82. The mouth of Arkansas River to Lake Itasca? *1,769.*
83. St. Louis to Rock Island?
84. Burlington to Memphis?
85. St. Paul to Vicksburg? *1,630.*
86. Quincy to Lake Itasca? *985.*
87. New Orleans to the Falls of St. Anthony? *2,039.*
88. Rock Island to Baton Rouge?
89. St. Louis to New Orleans?



90. If I owe $3,496, and I pay $1,748, how much do I then owe? *$1,748.*

91. What is the difference between 9,417 and 3,584? *5,833.*

92. The greater of two numbers is 11,419, and the less is 7,255. What is their difference? *4,164.*

93. The running expenses of a machine-shop for a year were $20,456, and the sales amounted to $31,217. What were the net earnings for the year? *$10,761.*

REVIEW PROBLEMS.

1. In the year 1860 there were 34 States in the Union, of which 11 seceded. How many States did not secede? *23.*

2. A farmer drew to market seven loads of hay, which weighed 1,577 pounds, 1,891 pounds, 1,648 pounds, 2,154 pounds, 1,736 pounds, 1,954 pounds, and 2,026 pounds. What was the weight of the seven loads?

3. On board an ocean steamer were 114 cabin passengers, 649 steerage passengers, and a crew of 87 persons. What was the whole number of persons on board? *850.*

4. A father and his two sons earned $1875 in a year, the elder son earning $638, and the younger son $459. How much did the father earn? *$778.*

5. A gardener received $218 for cabbages, and $439 for tomatoes. The expense of raising the cabbages was $84, and of the tomatoes $124. What were his profits on the two crops? *$449.*

6. A regiment when it entered the service mustered 1004 men; during the war 37 of these were killed, 48 died, 53 were taken prisoners, and 597 were discharged. How many men served through their term of enlistment? *269.*

7. In five successive weeks 79,747 tons, 84,324 tons, 68,953 tons, 76,081 tons, and 81,168 tons of coal were taken to Philadelphia by the Philadelphia and Reading Railroad. How many tons were carried over the road in the five weeks?

8. A contractor furnished 10,000 overcoats for the army, but 1,715 of them were condemned as imperfect. How many of them were accepted?

9. On the first of January an edition of 5,000 copies of a book was published. In January 396 copies were sold, in February 741, in March 1,214, in April 927, in May 643, and in June 584. How many copies remained unsold July 1? *495.*

10. A man was born in the year 1799, and died at the age of 67 years. In what year did he die? *In the year 1866.*

11. The battle of Lexington was fought in the year 1775, and President Lincoln was assassinated in 1865. How many years between the two events? *90.*

12. In the year 1861 the senior class of a certain college contained 46 students, the junior class 38, the sophomore class 59, and the freshman class 74. Of these, 12 seniors, 14 juniors, 18 sophomores, and 25 freshmen enlisted. How many students enlisted?

13. How many students remained in each class? How many remained in college? *148 remained in college.*

14. A railroad train left Cincinnati for St. Louis with 435 passengers. On the route 215 passengers left the cars, and 194 went aboard. How many were on the train when it reached St. Louis? *414.*

15. One year a merchant's sales amounted to $37,496. His goods cost $25,267, and his store expenses were $6,485. How much were his profits? *$5,744.*

16. One day 48,325 letters were received at the New York Post-office. Of these, 21,259 were directed to places in the State of New York, 20,048 to places in other states, and the rest to places in foreign countries. How many were directed to foreign countries? *7,018.*

17. At the beginning of the year A had property worth $2,350, but he owed $476. During the year he earned $1,156, and expended $879. How much was he worth at the end of the year? *$2,151.*

18. 1,153 − 967 + 10,000 − 5,308 = how many?

19. What is the difference between 13,647 + 2,593 − 6,483 and 20,000 − 4,931 + 5,006 − 7,285? *3,033.*

SECTION IV.

MULTIPLICATION.

INDUCTION.

(See Manual, page 216.)

39. 1. In this picture are 3 stems of a rose-bush, with 3 roses on each stem. 3 roses and 3 roses and 3 roses are how many roses? Then, 3 times 3 roses are how many roses?

2. On each of the stems are 4 buds. 4 buds and 4 buds and 4 buds are how many buds? 3 times 4 buds are how many?

3. On the branches of a cherry-tree we see 5 clusters, with 3 cherries in each cluster. 3 cherries + 3 cherries + 3 cherries + 3 cherries + 3 cherries are how many cherries? 5 times 3 cherries are how many?

4. On each of the 5 branches are 4 leaves. 5 times 4 leaves are how many leaves?

5. A boy at play with his blocks places them in rows on a table. If he counts them one way he has 3 rows, and 7 blocks in each row; but if he counts them another way he has 7 rows, and 3 blocks in each row. How many blocks has he?

How many are

6. 3 times 7 blocks? | 7. 7 times 3 blocks?

8. If a woman can weave 6 yards of rag carpet in 1 day, how many yards can she weave in 4 days?

9. A shoemaker can make 5 pairs of children's shoes in 1 day. How many pairs can he make in 8 days?

10. How much will 5 oranges cost, at 8 cents a piece?

The sum of $7 + 7 + 7 + 7 + 7$ is 35, and 5 times 7 are 35. By each of these methods we have found the sum of five 7's, or of as many 7's as there are ones in 5; but the second method is shorter than the first.

40. The process of finding, by a method shorter than Addition, the sum of as many times one number as there are ones in another, is *Multiplication.*

41. The result thus found is the *Product,* and

42. The numbers themselves are *Factors.*

43. The factor which is to be taken any certain number of times is the *Multiplicand,* and

44. The factor which shows how many times the multiplicand is to be taken, is the *Multiplier.*

The number of ones in the product must equal the number of ones of the multiplicand taken as many times as there are ones in the multiplier.

11. What is the product of 6 times 8 loaves of bread?

12. What is the product of the factors 10 and 7?

13. 7 times 8 are 56. Which of these numbers is the multiplicand? Which is the multiplier? Which is the product? Which are the factors?

45. This sign $\times$, written between two numbers, signifies that they are to be multiplied together.

It is called the *Sign of Multiplication,* and is read *times,* or *multiplied by.* Thus,

$6 \times 9 = 54$ may be read 6 times 9 equal 54, 6 times 9 are 54, 6 multiplied by 9 equal 54, or 6 multiplied by 9 are 54.

14. Read $7 \times 12 = 84$.

15. Read $4 \times 25 = 100$.

16. Read $3 \times 8 = 4 \times 6$.

17. Read $3 \times 4 \times 5 = 60$.

18. 6×7 plums are how many plums?

19. 10 × 4 apples = how many apples?

20. 3 × 2 × 5 balls = how many balls?

46. Numbers applied to objects or things are *Concrete Numbers;* as, 4 apples, 19 men, 237 books.

47. Numbers not applied to objects or things are *Abstract Numbers;* as, 4, 19, 237. (See Manual, page 216.)

48. MULTIPLICATION TABLE.

1	0	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9
2	0	1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2	2	2
	0	2	4	6	8	10	12	14	16	18
3	0	1	2	3	4	5	6	7	8	9
	3	3	3	3	3	3	3	3	3	3
	0	3	6	9	12	15	18	21	24	27
4	0	1	2	3	4	5	6	7	8	9
	4	4	4	4	4	4	4	4	4	4
	0	4	8	12	16	20	24	28	32	36
5	0	1	2	3	4	5	6	7	8	9
	5	5	5	5	5	5	5	5	5	5
	0	5	10	15	20	25	30	35	40	45
6	0	1	2	3	4	5	6	7	8	9
	6	6	6	6	6	6	6	6	6	6
	0	6	12	18	24	30	36	42	48	54
7	0	1	2	3	4	5	6	7	8	9
	7	7	7	7	7	7	7	7	7	7
	0	7	14	21	28	35	42	49	56	63
8	0	1	2	3	4	5	6	7	8	9
	8	8	8	8	8	8	8	8	8	8
	0	8	16	24	32	40	48	56	64	72
9	0	1	2	3	4	5	6	7	8	9
	9	9	9	9	9	9	9	9	9	9
	0	9	18	27	36	45	54	63	72	81
10	0	1	2	3	4	5	6	7	8	9
	10	10	10	10	10	10	10	10	10	10
	0	10	20	30	40	50	60	70	80	90

ORAL EXERCISES.

1.—1. Count by 2's to 20, in this manner: One 2 is 2, or once 2 is 2; two 2's are 4, or 2 times 2 are 4; three 2's are 6, or 3 times 2 are 6; four 2's are 8, or 4 times 2 are 8; and so on.

2. Multiply from 0 times 2 to 10 times 2, thus; 0 times 2 is 0, once 2 is 2, 2 times 2 are 4, 3 times 2 are 6, 4 times 2 are 8, and so on.

3. Multiply from 10 times 2 to 0 times 2, thus; 10 times 2 are 20, 9 times 2 are 18, 8 times 2 are 16, 7 times 2 are 14, and so on.

2.—1. Count by 3's to 30, in this manner: One 3 is 3, or once 3 is 3; two 3's are 6, or 2 times 3 are 6; three 3's are 9, or 3 times 3 are 9; four 3's are 12, or 4 times 3 are 12, and so on.

2. Multiply from 0 times 3 to 10 times 3, thus; 0 times 3 is 0, once 3 is 3, 2 times 3 are 6, 3 times 3 are 9, and so on.

3. Multiply from 10 times 3 to 0 times 3, thus; 10 times 3 are 30, 9 times 3 are 27, 8 times 3 are 24, and so on.

3.—1. Count by 4's to 40, thus: One 4 is 4, or once 4 is 4; two 4's are 8, or 2 times 4 are 8; three 4's are 12, or 3 times 4 are 12, and so on.

2. Multiply from 0 times 4 to 10 times 4, thus; 0 times 4 is 0, once 4 is 4, 2 times 4 are 8, 3 times 4 are 12, and so on.

3. Multiply from 10 times 4 to 0 times 4, thus; 10 times 4 are 40, 9 times 4 are 36, 8 times 4 are 32, and so on.

4.—1. Count by 5's to 50. (See Manual, page 216.)
2. Multiply from 0 times 5 to 10 times 5.
3. Multiply from 10 times 5 to 0 times 5.

5.—1. Count by 6's to 60.
2. Multiply from 0 times 6 to 10 times 6.
3. Multiply from 10 times 6 to 0 times 6.

6.—1. Count by 7's to 70.
2. Multiply from 0 times 7 to 10 times 7.
3. Multiply from 10 times 7 to 0 times 7.

7.—1. Count by 8's to 80.
2. Multiply from 0 times 8 to 10 times 8.
3. Multiply from 10 times 8 to 0 times 8.

8.—1. Count by 9's to 90.
2. Multiply from 0 times 9 to 10 times 9.
3. Multiply from 10 times 9 to 0 times 9.

9.—1. Count by 10's to 100.
2. Multiply from 0 times 10 to 10 times 10.
3. Multiply from 10 times 10 to 0 times 10.

CASE I.

The Multiplier One Figure.

FIRST METHOD.

49. Ex. 1. 37 + 37 + 37 + 37, or 4 times 37, are how many?

FIRST SOLUTION.
By Addition.

37
37
37
37
2
148

EXPLANATION.—In adding these numbers, we first find the sum of 7 ones taken 4 times, which is 28. We write the 8 ones of this sum as the ones of the final result, and the 2 tens we write in tens' place between the two parallel lines. We next find the sum of 3 tens taken 4 times, which is 12 tens; and adding to this sum the 2 tens of the first result, we write the 14 tens, which equal 4 tens and 1 hundred, as the tens and hundreds of the final result.

In the second solution we write the 37 only once, and as this number is to be taken 4 times, we write 4 under the right-hand figure. 7 ones + 7 ones + 7 ones + 7 ones, or 4 times 7 ones, are 28 ones. We write the 8 ones of this sum or product as the ones of the final result; and the 2 tens, which are to be added to the sum or product of the tens, we write in tens' place between the two parallel lines. Then, 3 tens + 3 tens + 3 tens + 3 tens, or 4 times 3 tens, are 12 tens; and adding to this sum or product the two tens of the first result, we write the 14 tens as the tens and hundreds of the final result.

SECOND SOLUTION.
By Multiplication.

37
4
2
148

(See Manual, page 216.)

Ex. 2. What is the product of 3,794 multiplied by 6?

EXPLANATION.—We write the multiplier under the multiplicand, and commence at the right to multiply. 6 × 4 ones = 24 ones, or 4 ones and 2 tens. We write the 4 ones for the ones of the final product, and the 2 tens in tens' place between the parallel lines. 6 × 9 tens = 54 tens, and 54 tens + 2 tens = 56 tens, or 6 tens and 5 hundreds. We write the 6 tens for the tens of the final product, and the 5 hundreds in hundreds' place between the parallel lines. 6 × 7 hundreds = 42 hundreds, and 42 hundreds + 5 hundreds = 47 hundreds, or 7 hundreds and 4 thousands. We write the 7 hundreds for the hundreds of the final product, and the 4 thousands in thousands' place between the parallel lines. 6 × 3 thousands = 18 thousands, and 18 thousands + 4 thousands = 22 thousands, or 2 thousands and 2 ten-thousands, and these we write for the thousands and ten-thousands of the final product.

SOLUTION.

3794 Multiplicand.
6 Multiplier.
452
22764 Product.

PROBLEMS.

1. How much will 3 tons of hay cost, at $12 a ton? *$36.*

2. In 1 day there are 24 hours. How many hours in 4 days? *96.*

3. How much will 4 pounds of raisins cost, at 22 cents a pound? *88 cents.*

4. If a canal-boat goes 42 miles in a day, how many miles will it go in 6 days? *252.*

5. A farm laborer worked 5 months for $16 a month. How much did his wages amount to? *$80.*

6. What is the product of 7×74? *518.*

7. The multiplicand is 96, and the multiplier is 8. What is the product? *768.*

8. A livery-man bought 3 horses, at $125 apiece. How much did they cost him?

9. How much will it cost for a party of 6 persons to go from New York to Liverpool on an ocean steamer, if the fare is $137? *$822.*

10. A music dealer sold 7 pianos, at $325 each. How much did he receive for all of them? *$2,275.*

11. How many quarts of milk will be used in a hotel in a week, if 36 quarts are used each day?

12. The factors are 2,147 and 5. What is the product?

13. A cooper sent to the mill 9 loads of flour-barrels, and each load contained 146 barrels. How many barrels did he send?

14. In one mile there are 5,280 feet. How many feet in 8 miles?

15. The Pennsylvania Central Railroad Company bought 6 locomotives, at $28,675 each. How much did they all cost? *$172,050.*

16. How much will 7 bushels of potatoes cost, at 56 cents a bushel? *392 cents.*

17. A farmer raised 8 acres of wheat, and each acre produced 25 bushels. How many bushels of wheat did he raise?

SECOND METHOD.

50. Ex. Multiply 473 by 9.

SOLUTION.

473
9
4,257

EXPLANATION.—We multiply each figure of the multiplicand by the multiplier, as in the First Method. 9×3 ones are 27 ones, or 7 ones and 2 tens. We write the 7 ones as the ones of the product, but instead of writing down the tens' figure, 2, we reserve it in the mind, to be added to the product of the tens. 9×7 tens are 63 tens, and 63 tens + 2 tens = 65 tens, or 5 tens and 6 hundreds. We write the 5 tens as the tens of the product, and reserve the 6 hundreds in the mind, to be added to the product of the hundreds. 9×4 hundreds are 36 hundreds, and 36 hundreds + 6 hundreds = 42 hundreds, or 2 hundreds and 4 thousands, and these we write as the hundreds and thousands of the product.

This is the method generally used.

PROBLEMS.

18. A druggist sold 8 gallons of kerosene, at 85 cents a gallon. How much did he receive for it? *680 cents.*

19. The distance from New York to Washington is 226 miles. How many miles does a man travel, who goes from New York to Washington and back? *452.*

20. The factors are 9 and 147. What is the product?

21. How many gallons in 6 hogsheads, each containing 124 gallons? *744.*

22. A market gardener bought 5 acres of land, at $635 per acre. How much did the land cost him?

23. Nine cars of a freight train are loaded with flour, and each car contains 96 barrels. How many barrels of flour on the train? *864.*

24. How many pounds in 6 barrels of Onondaga salt, each barrel containing 256 pounds?

25. How much will 122 pairs of boots cost, at $8 a pair?

51. In the picture on page 45 we see that 7 times 3 blocks are the same as 3 times 7 blocks; or that the product is the same, whichever of the two numbers is taken for the multiplicand.

In problem 25, \$8 is the *true multiplicand*, because, if one pair of boots costs \$8, 122 pairs will cost 122 times \$8; but since 122 times 8 is the same as 8 times 122, in solving the problem we may place 122 as the multiplicand, and use 8 as the multiplier.

52. *General Principles of Multiplication.*

I. *In the solution of problems, either factor may be used as the multiplicand.*

II. *The true multiplicand is that factor which would be used in solving the problem by Addition.*

III. *The multiplicand may be either an abstract or a concrete number.*

IV. *The multiplier must always be an abstract number.*

V. *The product is always of the same kind as the true multiplicand.*

(See Manual, page 216.)

PROBLEMS.

26. Thomas attended public school 76 days, and his tuition was 3 cents a day. How much was his school bill? *228 cts.*

27. In one week a newsboy sold 246 papers, at 5 cents each. How much did he receive for them?

28. A man's family expenses are \$4 a day. How much are they for a year, or 365 days? *\$1,460.*

29. How much will 2,755 army blankets cost, at \$2 apiece?

30. How many pounds in 6 bales of cotton, each weighing 478 pounds? *2,868.*

31. What will be the cost of building a horse railroad 4 miles long, at \$12,678 a mile? *\$50,712.*

32. How many gallons are 8 times 27,645 gallons?

33. How many pounds are 5 times 32,051 pounds?

34. What is the product of 6 times 1,026,348? *6,158,088.*

CASE II.

The Multiplier any number of Tens, Hundreds, Thousands, and so on.

53. Ex. Multiply 254 by 10.

SOLUTION.

254
10
2540

EXPLANATION. — We write the numbers as shown in the solution, and multiply each figure of the multiplicand by the multiplier, as in Case I. 10×4 ones $= 40$ ones; 10×5 tens $= 50$ tens, and 50 tens $+ 4$ tens $= 54$ tens; 10×2 hundreds $= 20$ hundreds, and 20 hundreds $+ 5$ hundreds $= 25$ hundreds.

The figures of the product are the same as those of the multiplicand, with a cipher on the right. Hence,

54. *Annexing a cipher to any number multiplies it by* 10.

55. *Annexing a second cipher multiplies by* 10 *again; that is, annexing two ciphers to any number multiplies it by* 10 *times* 10, *or* 100.

56. *Annexing three ciphers to a number multiplies it by* 10 *times* 100, *or* 1,000.

57. *Annexing four ciphers multiplies by* 10,000; *annexing five ciphers, by* 100,000; *and annexing six ciphers, by* 1,000,000.

PROBLEMS.

35. How much will 10 barrels of mess pork cost, at $23 a barrel? *$230.*

36. In constructing a telegraph line 100 miles long, how many poles will be required, allowing 16 poles to the mile?

37. How many panes of 8 by 12 glass in 1,000 boxes, each box containing 75 panes? *75,000.*

38. In one barrel of flour there are 196 pounds. How many pounds in 10,000 barrels? *1,960,000.*

39. Multiply 5,675 yards by 100,000. *567,500,000 yards.*

40. What is the product of 393 pounds multiplied by 1,000,000?

58. Ex. Multiply 254 by 30.

EXPLANATION.—30, or 3 tens = 3 times 10 ones, or 10 times 3 ones. Hence, 30 times 254 are 10 times 3 times 254. We may therefore multiply by 3, as in Case I., and the product thus obtained by 10. The final result, 7,620, is 10 times 3 times 254, or 30 times 254.

In the second solution, after multiplying 254 by 3, we have annexed a cipher to the result.

FIRST SOLUTION.

254
3
762
10
7620

SECOND SOLUTION.

254
30
7620

59. *To multiply by* 300, *we multiply by* 3, *and annex two ciphers to the product; to multiply by* 3,000, *we multiply by* 3, *and annex three ciphers, and so on.*

PROBLEMS.

41. At $65 a hogshead, how much will 300 hogsheads of molasses cost? *$19,500.*

42. How many bushels of oats will be required to keep 800 cavalry horses a year, if 187 bushels are required for each horse?

43. How many sheets of paper in 3,000 copies of Worcester's Dictionary, there being 112 sheets in each copy? *336,000.*

44. A man bought a farm of 70 acres, at $125 an acre. How much did the farm cost him? *$8,750.*

45. How many pounds of beef in 214 barrels, each barrel containing 200 pounds? *42,800.*

46. A railroad company bought 9,000 cords of wood, at $5 a cord. How much did the wood cost them?

47. A vessel at New York took on a cargo of 7,000 barrels of kerosene. If each barrel contained 43 gallons, how many gallons of kerosene in the cargo? *301,000.*

48. A wholesale grocer bought 500 chests of tea, each containing 56 pounds. How many pounds did all the chests contain? *28,000.*

49. In one hour there are 60 minutes. How many minutes in 1 day, or 24 hours? *1,440.*

50. How many pounds of cotton in 40,000 bales, each weighing 394 pounds? *15,760,000.*

51. An Illinois farmer had 80 acres of corn, which yielded 94 bushels an acre. How many bushels had he in the whole crop? *7,520.*

52. Multiply 249 by 4,000,000. *996,000,000.*

53. What is the product of 600,000 times 972?

54. The factors are 90,000 and 2,165. What is the product?

CASE III.

The Multiplier more than One Figure.

FIRST METHOD.

60. Ex. Multiply 563 by 34.

FIRST SOLUTION.

Multiplying by 4.	*Multiplying by 30.*	*Adding partial products.*
563	563	2252
4	30	16890
2252	16890	19142

EXPLANATION.—As 34 consists of 4 ones and 3 tens, or of 4 and 30; and as we can not multiply 563 by the whole 34 at once, we first multiply it by 4 and then by 30, and afterward add the two results or partial products. 19,142, the result thus obtained, is the sum of 4×563 plus 30×563, or 34×563.

In the First Solution, each of the three steps stands by itself; in the Second Solution they are placed together.

SECOND SOLUTION.

563
34
2252 } Partial
16890 } Products.
19142

In multiplying by 3 tens, or 30, we may first write a cipher in ones' place, and then to the left of it write the product obtained by multiplying by 3. In this manner solve the following problems.

PROBLEMS.

55. How many yards in 24 pieces of sheeting, each containing 42 yards? *1,008.*

56. How many pounds in a load of 74 bushels of oats, allowing 32 pounds to a bushel? *2,368.*

57. How much are 132 ounces of gold worth, at $16 an ounce?

58. How many hills of corn in a field that contains 36 rows, and 185 hills in each row?

59. A wholesale dealer in watches sold 48 watches, at $125 each. How much did he receive for them? *$6,000.*

60. Twelve things make a dozen. How many eggs in a barrel containing 87 dozens? *1,044.*

61. A western township contains 36 square miles. How many square miles in a county of 25 townships? *900.*

62. How many ounces of silver can be obtained from 392 tons of ore, at the rate of 34 ounces per ton.

SECOND METHOD.

61. Ex. What is the product of 243 times 2,156?

EXPLANATION.—In the First Solution, the ciphers on the right of the second and third partial products serve merely to fill the places of ones and tens; and since the sum of any number of 0's is 0, we really omit these ciphers in adding the partial products.

FIRST SOLUTION.

2156
243
6468
86240
431200
523908

We may omit to write these ciphers in the partial products, as shown in the Second Solution, without affecting the total product. By this method, the second partial product may be found by multiplying by 4 instead of 40; and the third, by multiplying by 2 instead of 200. But we must always

SECOND SOLUTION.

2156
243
6468
8624
4312
523908

62. *Write the first figure of each partial product directly under the figure of the multiplier used to obtain it.*

This is the method in general use.

In reading each partial product, its true value should be given. This is done by reading it as though the ciphers were written.

PROBLEMS.

63. A drover bought 13 young cattle, at $34 a head. How much did they cost him? *$442.*

64. An iron-founder bought 37 tons of pig-iron, at $45 a ton. How much did he pay for it? *$1,665.*

65. A merchant bought 26 pieces of calico, each containing 39 yards. How many yards in all? *1,014.*

66. How many miles will a railroad train run in 48 hours, running at the rate of 34 miles an hour? *1,632.*

67. How much must I pay for a farm of 67 acres, at $75 an acre?

68. A tobacco grower raised 10 acres of tobacco, which yielded 1,654 pounds to the acre. How much did his tobacco crop weigh? *16,540 pounds.*

69. What is the product of 58 times 157? *9,106.*

70. 73 × 593 = how many? *43,289.*

71. 84 × 748 square miles = how many square miles?

(72)	(73)	(74)	(75)	(76)
4,306	2,087	8,009	10,609	50,028
284	659	573	138	971

63. Ex. Multiply 6,149 by 503.

EXPLANATION.—In the Second Solution we have multiplied first by the 3 (ones), and then by the 5 (hundreds), omitting to multiply

FIRST SOLUTION.	SECOND SOLUTION.
6149	6149
503	503
18447	18447
0000	30745
30745	3092947
3092947	

by the 0 (tens), because 0 times 6,149 is 0, as shown in the First Solution.

64. *Since* 0 *times any number is* 0, *we may always omit to multiply by ciphers that stand between other figures in the multiplier, being careful to write the first figure of each partial product directly under the figure of the multiplier used to obtain it.*

PROBLEMS.

77. At the close of the war, in 1865, a quarter-master sold 905 mules, at $183 apiece. How much was received for them? *$165,615.*

78. In 1864, a certain county raised 706 men for the army, paying a bounty of $855 to each man. How much bounty money was paid by the county? *$603,630.*

79. At the office of a daily newspaper, 217 reams of paper are used each day. How many reams will be required to last 308 days?

80. What will be the weight of the wire for a line of telegraph 207 miles long, if one mile of wire weighs 326 pounds? *67,482 pounds.*

81. In building a factory a mason used 409 loads of brick, each load containing 648 bricks. How many bricks did he use? *265,032.*

82. The daily wages of the hands in a cotton-factory amount to $736. How much is that for a year of 307 working-days?

83. Multiply 17,248 by 6,003. *103,539,744.*

84. Find the product of 53,276 multiplied by 5,002.

65. Ex. What is the product of 4,300 times 2,394?

EXPLANATION. — To multiply by 300, we first multiply by 3, and then annex two ciphers; (Case II). In the same manner, to multiply by 4,300 we first multiply by 43, and then annex two ciphers, as shown in the solution.

SOLUTION.

2394
4300
7182
9576
10294200

66. *Rule for Multiplication of Integers.*

I. *Write the greater number for the multiplicand, and under it the less number for the multiplier.*

II. When the multiplier consists of only one figure, *Commence with the ones, and multiply successively each figure of the multiplicand by the multiplier. Write in the product the right-hand figure of each result, and add the left-hand figure to the next result, as in Addition.*

III. When the multiplier consists of more than one figure,

1. *Commence at the right, multiply the multiplicand by each figure of the multiplier, except ciphers, and place the right-hand figure of each partial product under that figure of the multiplier used to obtain it.*

2. *Add the partial products.*

IV. When the multiplier ends with one or more ciphers,

After multiplying by the other figures, annex an equal number of ciphers to the product.

PROBLEMS.

85. A pork packer bought 2,400 sacks of Turks' Island salt, each sack containing 168 pounds. How many pounds of salt did he buy? *403,200.*

86. A railroad company bought 5,800 bars of railroad iron, each bar weighing 402 pounds. How much did the whole weigh? *2,331,600 pounds.*

87. A whaler entered New Bedford with a cargo of 1,760 barrels of whale oil, each barrel containing 39 gallons. How many gallons of oil in the cargo? *68,640.*

88. How much would the yearly pay of an army of 650,000 men amount to, at $192 per man? *$124,800,000.*

89. If the factors are 9,654 and 21,800, what is the product?

90. How many cords of wood can be cut from 85 acres of wood-land, at the rate of 65 cords to the acre?

91. How many barrels will be made at a barrel factory in 56 days, if 159 barrels are made each day? *8,904.*

92. How much will 94 passenger cars cost, at $2,475 each?

93. If 97 tons of railroad iron are required for one mile of track, how many tons will be required for a road 359 miles long? *34,823.*

94. An agent in one year sold 795 sewing machines, at $65 apiece. How much were his receipts? *$51,675.*

95. The number of trips made by the boats at a certain ferry is 234 per day, and the average number of passengers per trip is 108. How many persons cross the ferry in a day? *25,272.*

96. Multiply 1,327 by 246. *326,442.*

97. What is the product of the factors 2,076 and 382?

67. Ex. The factors are 5,800 and 27. What is the product?

EXPLANATION.—In the First Solution we have multiplied 27 by 5,800, by first multiplying by 58, and to the product annexing two ciphers. But as 58 times 27 = 27 times 58, (Prin. I.), in the Second Solution we have multiplied 58 by 27, and to the product annexed two ciphers. The results in the two solutions are alike, and either method may be employed.

FIRST SOLUTION.

27
5800

216
135

156600

SECOND SOLUTION.

5800
27

406
116

156600

PROBLEMS.

98. In the year 1865 there had been 16 Presidents, whose united terms of office amounted to 76 years. How much had their salaries amounted to, at $25,000 a year? *$1,900,000.*

99. At the rate of 1,400 words an hour, how many words can be sent over a telegraph line in 24 hours? *33,600.*

100. How much will a clergyman's salary amount to in 17 years, at $1,200 a year? *$20,400.*

101. There are 5,280 feet in a mile. How many feet long is the Pennsylvania Railroad track, which is 353 miles long?

102. The multiplicand is 39,200, and the multiplier is 65. What is the product? *2,548,000.*

103. Multiply 58,000 by 73. *4,234,000.*

104. 80,004 × 12,357 = how many? *988,609,428.*

105. At a saw-mill 185 logs were sawed into boards, each log making 642 feet of lumber. How many feet did all of them make?

106. If 1,594 pounds of saltpeter are used in making 1 ton of gunpowder, how many pounds will be used in making 1,056 tons?

68. Ex. Multiply 67,000 by 2,100.

EXPLANATION.—We first multiply 67,000 by 21, as before explained, and to 1,407,000, the product thus obtained, we annex two ciphers; or to the product of 21 × 67 we annex three ciphers for those at the right of the 67 (thousands) in the multiplicand, and two for those at the right of the 21 (hundreds) in the multiplier. (See Manual, page 217.)

SOLUTION.

67000
2100
67
134
140700000

PROBLEMS.

107. One year a piano-forte manufacturer made 360 pianos, at a cost of $270 each. How much did they all cost him? *$97,200.*

108. How much will it cost to build 80 miles of turnpike road, at $1,500 a mile? *$120,000.*

109. If a rope-maker can spin 3,700 feet of rope in a day, how many feet can he spin in 260 days? *962,000.*

110. A barge was loaded with 600 bales of hay, which weighed 290 pounds each. What was the whole weight of the hay? *174,000 pounds.*

111. What is the product of 63,000 times 1,850? *116,550,000.*

112. A manufacturer of reapers and mowers sold in one year 2,476 machines, at $125 each. How much did he receive for them? *$309,500.*

113. How many poles will be required for a hop-yard of 40 acres, allowing 1,280 poles to the acre?

114. A common clock strikes 156 times every day. How many times does it strike in one year, or 365 days?

115. The three factors of a number are 39, 24, and 17. What is the number? *15,912.*

69. When a final product is obtained by the use of more than two factors, the process is called *Continued Multiplication.* (See Manual, page 217.)

116. A grocer packed 23 barrels of eggs, putting 83 dozen in each barrel. How many eggs did he pack? *22,908.*

117. How many sheets in 45 reams of letter paper, there being 20 quires in each ream, and 24 sheets in each quire?

118. How many miles will a conductor on a railroad 81 miles long travel in 13 years, if he makes a round trip every day for 300 days each year? (See Manual, page 217.) *631,800 miles.*

119. In 7 dozen papers of pins, how may pins, each paper containing 10 rows, and each row 36 pins? *30,240.*

120. What number is that of which the factors are 5,006, 79, and 840?

REVIEW PROBLEMS.

(See Manual, page 217.)

1. A mechanic earns $45 a month, and his expenses are $476 a year. How much can he save in one year, or 12 months? *$64.*

2. A stock train consists of 19 cars, each containing 17 cattle. How much do all the cattle weigh, their average weight being 895 pounds?

3. A man bought a house and lot, for which he paid $1,275 down, and $250 each year for 7 years. How much did the place cost him? *$3,025.*

4. When Victor was 15 years old he had $100. He saved $37 each year until he was 21. How much money had he then? *$322.*

5. A book-keeper earns $150 a month, and expends $68. How much does he save in a year? *$984.*

6. An agent sold 38 sets of the New American Cyclopædia, each set containing 16 volumes, at $5 a volume. How much did he receive for them?

7. A steamer was burned at sea, having on board 296 passengers and a crew of 73 persons. Of these, 117 passengers and 49 of the crew were saved. How many persons were lost? *203.*

8. A hotel keeper charges $5 a week for board. How much will he receive from 27 boarders in one year, or 52 weeks?

9. A merchant tailor bought 84 pieces of broadcloth, each piece containing 37 yards, at $6 a yard. How much did the cloth cost him?

10. If 33 pickets are used in building one rod of fence, how many pickets will be required for the fence around a lot 16 rods long and 15 rods wide? *2,046.*

11. How much will a baker receive for 48 pounds of crackers, at 14 cents a pound, and 128 loaves of bread, at 9 cents a loaf?

12. A provision dealer bought 850 barrels of pork, at $17 a barrel, and sold the lot for $696 more than cost. How much did he receive?

13. A produce dealer bought 4,124 bushels of wheat, 2,476 bushels of corn, and 1,218 bushels of oats. After selling 3,684 bushels of wheat, 1,787 bushels of corn, and 975 bushels of oats, how many bushels of each had he left? *689 bu. of corn.*

14. A is worth $675; B is worth twice as much; and C is worth 4 times as much as both. How much are B and C each worth? How much are the three worth? *Worth of all, $10,125.*

15. A wholesale grocer, having 156 chests of tea, each containing 84 pounds, sold 57 chests to one customer, and 23 chests to another. How many chests, and how many pounds of tea had he left? *6,384 pounds.*

SECTION V.

DIVISION.

INDUCTION.

(See Manual, page 217.)

70. 1. Some lumbermen made a raft of 20 logs, putting 5 logs in each crib. Of how many cribs did the raft consist?

2. Of how many cribs will the raft consist if they use 25 logs?

3. How many cribs will there be in the raft if they use 30 logs?

4. To how many girls can I give 24 peaches, if I give 4 peaches to each girl?

5. One afternoon Robert found 21 chestnuts in burs, each bur containing 3 chestnuts. How many burs did he find?

6. In a raft of 4 cribs there are 20 logs. Of how many logs does each crib consist?

7. In a raft of 30 logs there are 6 cribs. How many logs in each crib?

8. A furniture dealer sold 4 rocking-chairs for $28. How many dollars did he receive for each of them?

9. A military company of 42 men marched in 6 equal files. How many men were in each file?

10. A grocer paid $32 for 8 barrels of apples. What was the price per barrel?

In solving each of the first five examples, we find how many times one of two numbers is contained in the other; and in solving each of the other examples, we separate one of two numbers into as many equal parts as there are ones in the other.

71. The process of finding how many times one of two numbers is contained in the other, or of finding one of the equal parts into which a number may be separated, is *Division.*

72. The result thus found is the *Quotient.*

73. The number to be divided is the *Dividend;* and

74. The number by which the dividend is to be divided, is the *Divisor.*

The number of ones in the dividend must be equal to the number of ones in the divisor taken as many times as there are ones in the quotient.

11. How many times 8 loaves of bread are 56 loaves?

12. What is the quotient of 63 divided by 7?

13. The dividend is 54, and the divisor is 6. What is the quotient?

75. When any number of things is divided into 2 equal parts, one of those parts is *one half* of that number of things. One of the 3 equal parts into which any number may be divided is *one third* of that number; one of the 4 equal parts is *one fourth;* one of the 5 equal parts is *one fifth*, and so on. (See Manual, page 217.)

76. One half of a number is found by dividing it by 2; one third by dividing it by 3; one fourth by dividing it by 4; one fifth by dividing it by 5; one sixth by dividing it by 6, and so on.

14. A man's farm contains 70 acres, and one seventh of it is woodland. How many acres of woodland has he?

15. If 9 table spreads cost \$36, what part of \$36 will 1 table spread cost? How many dollars will one cost?

16. How will you find one eighth of 48? In this question, what is the dividend? What is the divisor?

17. If 30 yards of carpeting be cut into 5 breadths, how many yards will there be in each breadth?

77. This sign $\div$, written between two numbers, signifies that the number before it is to be divided by the number after it.

It is called the *Sign of Division*, and is read *divided by*. Thus, $72 \div 6 = 12$ is read, 72 divided by 6 equals 12.

18. Read $99 \div 9 = 11$.

19. Read $64 \div 8 = 8$.

20. $81 \div 9 =$ how many?

21. $98 \div 7 = 14$. In this expression, which number is the dividend, which the divisor, and which the quotient?

22. A teamster feeds his horses 6 bushels of oats each week. How many weeks will 60 bushels last them?

23. If a teamster feeds his horses 60 bushels of oats in 10 weeks, how many bushels does he feed in 1 week?

(See Manual, page 217.)

78. DIVISION TABLE.

$\frac{0}{0}\ \frac{1}{1}\ \frac{2}{2}\ \frac{3}{3}\ \frac{4}{4}\ \frac{5}{5}\ \frac{6}{6}\ \frac{7}{7}\ \frac{8}{8}\ \frac{9}{9}$ ⌊ 1	$\frac{0}{0}\ \frac{6}{1}\ \frac{12}{2}\ \frac{18}{3}\ \frac{24}{4}\ \frac{30}{5}\ \frac{36}{6}\ \frac{42}{7}\ \frac{48}{8}\ \frac{54}{9}$ ⌊ 6
$\frac{0}{0}\ \frac{2}{1}\ \frac{4}{2}\ \frac{6}{3}\ \frac{8}{4}\ \frac{10}{5}\ \frac{12}{6}\ \frac{14}{7}\ \frac{16}{8}\ \frac{18}{9}$ ⌊ 2	$\frac{0}{0}\ \frac{7}{1}\ \frac{14}{2}\ \frac{21}{3}\ \frac{28}{4}\ \frac{35}{5}\ \frac{42}{6}\ \frac{49}{7}\ \frac{56}{8}\ \frac{63}{9}$ ⌊ 7
$\frac{0}{0}\ \frac{3}{1}\ \frac{6}{2}\ \frac{9}{3}\ \frac{12}{4}\ \frac{15}{5}\ \frac{18}{6}\ \frac{21}{7}\ \frac{24}{8}\ \frac{27}{9}$ ⌊ 3	$\frac{0}{0}\ \frac{8}{1}\ \frac{16}{2}\ \frac{24}{3}\ \frac{32}{4}\ \frac{40}{5}\ \frac{48}{6}\ \frac{56}{7}\ \frac{64}{8}\ \frac{72}{9}$ ⌊ 8
$\frac{0}{0}\ \frac{4}{1}\ \frac{8}{2}\ \frac{12}{3}\ \frac{16}{4}\ \frac{20}{5}\ \frac{24}{6}\ \frac{28}{7}\ \frac{32}{8}\ \frac{36}{9}$ ⌊ 4	$\frac{0}{0}\ \frac{9}{1}\ \frac{18}{2}\ \frac{27}{3}\ \frac{36}{4}\ \frac{45}{5}\ \frac{54}{6}\ \frac{63}{7}\ \frac{72}{8}\ \frac{81}{9}$ ⌊ 9
$\frac{0}{0}\ \frac{5}{1}\ \frac{10}{2}\ \frac{15}{3}\ \frac{20}{4}\ \frac{25}{5}\ \frac{30}{6}\ \frac{35}{7}\ \frac{40}{8}\ \frac{45}{9}$ ⌊ 5	$\frac{0}{0}\ \frac{10}{1}\ \frac{20}{2}\ \frac{30}{3}\ \frac{40}{4}\ \frac{50}{5}\ \frac{60}{6}\ \frac{70}{7}\ \frac{80}{8}\ \frac{90}{9}$ ⌊ 10

ORAL EXERCISES.

1.—1. Divide by 2 from 2 in 0 to 2 in 20, thus: 2 in 0, 0 times; 2 in 2, once; 2 in 4, 2 times, and so on.

2. Divide by 2 from 2 in 20 to 2 in 0, thus: 2 in 20, 10 times; 2 in 18, 9 times; 2 in 16, 8 times, and so on.

3. Divide by 2 from 1 half of 0 to 1 half of 20, thus: 1 half of 0 is 0; 1 half of 2 is 1; 1 half of 4 is 2, and so on.

4. Divide by 2 from 1 half of 20 to 1 half of 0, thus: 1 half of 20 is 10; 1 half of 18 is 9; 1 half of 16 is 8, and so on.

2.—1. Divide by 3 from 3 in 0 to 3 in 30, thus: 3 in 0, 0 times; 3 in 3, once; 3 in 6, 2 times, and so on.

2. Divide by 3 from 3 in 30 to 3 in 0, thus; 3 in 30, 10 times; 3 in 27, 9 times; 3 in 24, 8 times, and so on.

3. Divide by 3 from 1 third of 0 to 1 third of 30, thus; 1 third of 0 is 0; 1 third of 3 is 1; 1 third of 6 is 2, and so on.

4. Divide by 3 from 1 third of 30 to 1 third of 0, thus: 1 third of 30 is 10; 1 third of 27 is 9; 1 third of 24 is 8, and so on.

3.—1. Divide by 4 from 4 in 0 to 4 in 40, thus: 4 in 0, 0 times; 4 in 4, once; 4 in 8, 2 times, and so on.

2. Divide by 4 from 4 in 40 to 4 in 0, thus: 4 in 40, 10 times; 4 in 36, 9 times; 4 in 32, 8 times, and so on.

3. Divide by 4 from 1 fourth of 0 to 1 fourth of 40, thus: 1 fourth of 0 is 0; 1 fourth of 4 is 1; 1 fourth of 8 is 2, and so on.

4. Divide by 4 from 1 fourth of 40 to 1 fourth of 0, thus: 1 fourth of 40 is 10; 1 fourth of 36 is 9; 1 fourth of 32 is 8, and so on.

4.—1. Divide by 5 from 5 in 0 to 5 in 50. (See Manual, page 217.)

2. Divide by 5 from 5 in 50 to 5 in 0.

3. Divide by 5 from 1 fifth of 0 to 1 fifth of 50.

4. Divide by 5 from 1 fifth of 50 to 1 fifth of 0.

5.—1. Divide by 6 from 6 in 0 to 6 in 60.

2. Divide by 6 from 6 in 60 to 6 in 0.

3. Divide by 6 from 1 sixth of 0 to 1 sixth of 60.

4. Divide by 6 from 1 sixth of 60 to 1 sixth of 0.

6.—1. Divide by 7 from 7 in 0 to 7 in 70.

2. Divide by 7 from 7 in 70 to 7 in 0.

3. Divide by 7 from 1 seventh of 0 to 1 seventh of 70.

4. Divide by 7 from 1 seventh of 70 to 1 seventh of 0.

7.—1. Divide by 8 from 8 in 0 to 8 in 80.

2. Divide by 8 from 8 in 80 to 8 in 0.

3. Divide by 8 from 1 eighth of 0 to 1 eighth of 80.

4. Divide by 8 from 1 eighth of 80 to 1 eighth of 0.

8.—1. Divide by 9 from 9 in 0 to 9 in 90.
2. Divide by 9 from 9 in 90 to 9 in 0.
3. Divide by 9 from 1 ninth of 0 to 1 ninth of 90.
4. Divide by 9 from 1 ninth of 90 to 1 ninth of 0.

9.—1. Divide by 10 from 10 in 0 to 10 in 100.
2. Divide by 10 from 10 in 100 to 10 in 0.
3. Divide by 10 from 1 tenth of 0 to 1 tenth of 100.
4. Divide by 10 from 1 tenth of 100 to 1 tenth of 0.

CASE I.

The Divisor One Figure.

FIRST METHOD.

79. Ex. What is the quotient of 696 divided by 3.

FIRST SOLUTION.

696	3
600	200
96	30
90	2
6	232
6	

EXPLANATION.—Since the dividend is too large to be divided mentally, we place the divisor at the right of it, separating them by a line, and draw a line under the divisor to separate it from the quotient. Then commencing at the left hand, we divide each figure of the dividend by the divisor, thus; 3 is contained in 6 hundreds, 2 hundred times. We write this result below the divisor, for the first partial quotient. We have now used 600 (or 3 times 200) of the dividend; and, subtracting 600, we have a remainder of 96, or 9 tens and 6 ones, yet to be divided. 3 is contained in 9 tens or 90, 3 tens or 30 times. We write the 3 tens or 30 for the second partial quotient. We have now used 90 (90 tens), or 3 times 30; and, subtracting it from 96, the partial dividend, we have 6 ones for another partial dividend. 3 is contained in 6 ones, 2 times. We write the 2 (ones) as the third partial quotient, and subtract the 6 ones from the last partial dividend. We have now used all the figures of the dividend, and we have therefore found all the partial quotients. The

sum of these partial quotients, or 232, is the total quotient required.

SECOND SOLUTION.

$$\begin{array}{r|l} 696 & 3 \\ 6 & \overline{232} \\ \hline 9 & \\ 9 & \\ \hline 6 & \\ 6 & \\ \hline \end{array}$$

EXPLANATION. — We first divide the 6 (hundreds) of the dividend by 3, as before, and the quotient, 2 (hundreds), we write as the first figure of the final quotient. 3 times 2 (hundreds) are 6 (hundreds), and since this is to be subtracted from the dividend, we write it under the hundreds' figure of the dividend. 6 (hundreds) from 6 (hundreds) leaves 0. The next part of the dividend to be divided is the 9 (tens), which we bring down for a partial dividend. 3 is contained in 9 (tens), 3 (tens) times, and we write this 3 as the second figure of the final quotient. 3 times 3 (tens) are 9 (tens), and since this is to be subtracted from the tens of the dividend, we write it under the partial dividend. 9 (tens) from 9 (tens) leaves 0. The next part of the dividend to be divided is the 6 (ones), which we bring down for the last partial dividend. 3 is contained in 6 (ones), 2 (ones) times, and we write this 2 as the third figure of the final quotient. 3 times 2 (ones) are 6 (ones), and since this is to be subtracted from the ones of the dividend, we write it under the last partial dividend. 6 (ones) from 6 (ones) leaves 0. We have now used all the figures of the dividend, and the result, 232, is the quotient required.

In this solution we have taken the same steps as in the first, but we have omitted to write the ciphers of the partial quotients, because the value of each figure is known without them, and we thus use fewer figures. By writing the 2 (hundreds) in hundreds' place, the 3 (tens) in tens' place, and the 2 (ones) in ones' place, we really add 200, 30, and 2.

PROBLEMS.

1. At $4 apiece, how many hats can be bought for $848?

2. If 3 workmen can lay 3,396 bricks in a day, what part of 3,396 bricks can one man lay? How many bricks can one man lay? *1,132.*

3. Four newsboys bought 84 newspapers, each taking an equal share of them. How many papers did each boy take?

4. If 2 men can chop 28 cords of wood in a week, how many cords can one man chop?

5. A door maker received $639 for doors, at $3 each. How many doors did he sell? *213.*

6. At $4 a cord, how many cords of wood can be be bought for $1,648?

7. How many barrels of flour must a miller sell each day to sell 216 barrels in 3 days?

8. If a house painter earns $255 in 5 months, how much does he earn in one month? *$51.*

9. If 4 tons of coal are used each day in an iron-foundery, how many days will 3,284 tons last?

SOLUTION OF PROBLEM 6.

1648	4
16	412
4	
4	
8	
8	

80. Ex. Divide 917 by 7.

SOLUTION.

917	7
7	131
21	
21	
7	
7	

EXPLANATION.—Since 9 is greater than 7, 7 is contained in 9 at least 1 time; and since 2 times 7 are 14, and 14 is greater than 9, 7 is not contained in 9, 2 times. We therefore write the 1 as the first figure of the quotient. 7 times 1 (hundred) are 7 (hundreds); and since this is to be subtracted from the dividend, we write it under the hundreds' figure of the dividend. 7 (hundreds) from 9 (hundreds) leave 2 (hundreds). We next bring down the 1 (ten) at the right of the 2 (hundreds), and we have 21 (tens) for a partial dividend. 7 is contained in 21 (tens), 3 (tens) times, and this 3 we write as the second figure of the quotient. We complete the solution of the example, as before explained.

PROBLEMS.

10. Into how many fields of 8 acres each can a farm of 96 acres be divided? *12.*

11. A glazier set 966 panes of glass for a hotel, setting 6 panes in each sash. How many sashes did he use?

12. How many dollars must a man save each year, to save $968 in 4 years? *$242.*

13. In one season a farmer's wife made 855 pounds of butter from the milk of 5 cows. How many pounds of butter was that to each cow? *171.*

14. A man bought a farm for $7,248, and paid for it in 4 equal yearly payments. How much did he pay each year?

15. In one week there are 168 hours. How many hours in one day?

SOLUTION OF PROBLEM 15.

$$\begin{array}{r|l} 168 & 7 \\ \underline{14} & \overline{24} \\ 28 & \\ \underline{28} & \end{array}$$

16. How many miles must I drive my horse each day, to drive 185 miles in 5 days? *37.*

17. A livery-man paid $288 for 3 horses. How much did they cost him apiece? *$96.*

18. How many days will it take a girl to braid 1,084 straw hats, if she braids 4 hats each day?

19. An iron-founder sold 1,255 pounds of plowpoints, weighing 5 pounds each. How many plowpoints did he sell?

20. A coal dealer received $4,627 for coal, at $7 a ton. How many tons did he sell? *661.*

21. A man built a block of 8 dwellings at a cost of $9,888. How much was the cost of each?

SOLUTION OF PROBLEM 21.

$$\begin{array}{r|l} 9888 & 8 \\ \underline{8} & \overline{1236} \\ 18 & \\ \underline{16} & \\ 28 & \\ \underline{24} & \\ 48 & \\ \underline{48} & \end{array}$$

22. In one bushel there are 4 pecks. How many bushels in 6,548 pecks?

23. At a freight house are 222 casks of kerosene, to be loaded on 6 platform cars. How many casks must be loaded upon each car? *37.*

24. What is the quotient of 44,532 ÷ 6?

25. How many miles must a steam-ship run each day, to make a voyage of 1,771 miles in 7 days?

26. What is the quotient of 640,955 ÷ 7? *91,565.*

81. Ex. Divide 3,248 by 8.

FIRST SOLUTION.

$$\begin{array}{r|l} 3248 & 8 \\ \underline{32} & \overline{406} \\ 4 & \\ \underline{0} & \\ 48 & \\ \underline{48} & \end{array}$$

EXPLANATION. — 8 is contained in 32 (hundreds), 4 (hundreds) times. We write 4 for the first figure of the quotient, and multiplying by the divisor and subtracting, we have no remainder. The next figure to be divided is the 4 (tens) of the dividend, and we bring it down for a partial dividend. Since 8 is contained in 4 (tens), 0 (tens) times, we write the 0 for the second figure of the quotient. 8 times 0 (tens) are 0 (tens), and 0 (tens) subtracted from 4 (tens) leaves 4 (tens). We next bring down the 8 (ones), and we have 48 for a new partial dividend. We complete the solution, as before explained.

SECOND SOLUTION.

$$\begin{array}{r|l} 3248 & 8 \\ \underline{32} & \overline{406} \\ 48 & \\ \underline{48} & \end{array}$$

When we multiply the 0 (tens) of the quotient by the divisor and subtract, the remainder thus obtained is the same that we had before. We may therefore omit this multiplication and subtraction, as shown in the second solution.

82. *Whenever the partial dividend is less than the divisor, we place* 0 *in the quotient and bring down the next figure of the dividend for a new partial dividend.*

PROBLEMS.

27. How many days will it take a cooper to make 2,718 barrels, if he makes 9 barrels each day? *302.*

28. A farmer harvested 536 bushels of corn from 8 acres. What was the yield per acre? *67 bushels.*

29. In how many days can a teamster draw 2,187 loads of gravel, if he draws 9 loads each day?

30. Divide 25,041 by 3. *Quotient, 8,347.*

31. How much is one sixth of $16,236? *$2,706.*

32. If a tract of 1,435 acres of land be divided into 7 equal farms, how many acres will there be in each farm?

33. Divide 47,565 by 5. *Quotient, 9,513.*
34. Divide 2,751,075 by 5. *Quotient, 550,215.*
35. Divide 54,247,296 by 8. *Quotient, 6,780,912.*
36. What is the quotient of 78,462,729 ÷ 9? *8,718,081.*

SECOND METHOD.

83. Ex. What is the quotient of 4,368 ÷ 7?

SOLUTION.

$$\begin{array}{r|l} 4368 & 7 \\ \hline 624 & \end{array}$$

EXPLANATION.—We write the dividend and divisor as before, but below the dividend we draw a horizontal line, under which to write the quotient. 7 is contained in 43 (hundreds), 6 (hundreds) times. We write the 6 under the horizontal line, directly below the hundreds of the dividend, as the first figure of the quotient. 7 times 6 (hundreds) are 42 (hundreds). We subtract the 42 (hundreds) from the 43 (hundreds) of the dividend *mentally*, and to the remainder, 1 (hundred), we *mentally* unite the 6 (tens), making 16 (tens). 7 is contained in 16 (tens), 2 (tens) times. We write the 2 (tens) as the second figure of the quotient, and multiply the 2 (tens) by the divisor 7. The result, 14 (tens), we subtract *mentally* from the 16 (tens), and to the remainder, 2 (tens), we *mentally* unite the 8 (ones), making 28. 7 is contained in 28, 4 times, and this we write as the third figure of the quotient. We again multiply and subtract *mentally*, and we have no remainder.

(See Manual, page 217.)

In solving problems by this method, the several steps in the process are the same as by the First Method; but the multiplications and subtractions are performed mentally, and hence fewer figures are used.

84. When, in dividing, all the products and partial dividends are written, the process is *Long Division.*

85. When, in dividing, only the divisor, dividend, and quotient are written, the process is *Short Division.*

PROBLEMS.

37. A railroad company paid $6,828 for wood, at $6 a cord. How many cords did they buy? *1,138.*

38. How much is 1 fourth of 384 acres of land?

39. If 3 tons of hay will keep one horse through the winter, how many horses will 234 tons keep? *78.*

40. If a family use 8 pounds of sugar in one week, how many weeks will 416 pounds last them?

41. A farmer raised 1,892 bushels of turnips from 4 acres of land. How many bushels did the land yield per acre?

42. How many barrels of flour, at $8 a barrel, can be bought for $26,120? *3,265.*

43. Two men, owning a ship, sold it to 3 others for $13,050. How much did each seller receive, and how much did each buyer pay? *Each buyer paid $4,350.*

44. Divide 386,948 by 4. *Quotient, 96,737.*

45. Divide 99,627,342 by 6. *Quotient, 16,604,557.*

46. What is the quotient of 8,765,875 ÷ 5?

47. What is the quotient of 200,004,234 ÷ 6?

48. Divide $2,703,848 by 7. *Quotient, 386,264.*

49. The dividend is 3,474,963, and the divisor is 9. What is the quotient?

50. Divide 720,152,088 by 8. *Quotient, 90,019,011.*

CASE II.

The Divisor more than One Figure.

86. Ex. Divide 8,058 by 34.

SOLUTION.

$$\begin{array}{r|l} 8058 & 34 \\ 68 & \overline{237} \\ \hline 125 & \\ 102 & \\ \hline 238 & \\ 238 & \end{array}$$

EXPLANATION.—When the divisor consists of two or more figures, we solve the problem by long division. Since 34 is contained in 80, the first two figures of the dividend, 2 times, we place the 2 for the first figure of the quotient. 2 times 34 are 68, which subtracted from 80 leaves

12. Bringing down the 5 of the dividend, we have 125 for a partial dividend. 34 is contained in 125, 3 times, and we place the 3 for the next figure of the quotient. 3 times 34 are 102, which subtracted from 125, leaves 23. Bringing down the 8 of the dividend, we have 238 for a new partial dividend. 34 is contained in 238, 7 times, and we write the 7 for the third figure of the quotient. 7 times 34 are 238, which subtracted from the last partial dividend, leaves no remainder.

87. Sometimes we can not tell exactly how many times the divisor is contained in a partial dividend.

For example, divide 9,796 by 124.

We do not know how many times 124 is contained in 979, the first partial dividend, but we will suppose that it is contained 6 times. After multiplying and subtracting, we have a remainder of 235, which is greater than the divisor 124. Hence 124 is contained in 979 more than 6 times. Let us now suppose that it is contained 8 times. But 8 times 124 is 992, which is more than 979. Hence 124 is not contained in 979 as many as 8 times. We therefore conclude that 7 is the correct quotient figure.

FIRST TRIAL.

9796	124
744	6
235	

SECOND TRIAL.

9796	124
992	8

SOLUTION.

9796	124
868	79
1116	
1116	

88. From this example we learn that

I. *When any remainder is greater than the divisor, the quotient figure is too small.*

II. *When any product is greater than the partial dividend, the quotient figure is too great.* (See Manual, page 217.)

PROBLEMS.

51. A carpenter received $156 for building 12 rods of fence. How much did he receive a rod? *$13.*

52. How many weeks will it take a house painter to earn $168, if he earns $14 a week? *12.*

53. If a freight train runs 16 miles an hour, how many hours will it be in running 368 miles? *23.*

54. In one day there are 24 hours. How many days in 816 hours?

55. An oil well in Pennsylvania produced 5,394 barrels of oil in 31 days. How many barrels were produced daily?

56. A miller packed 13,475 pounds of flour in sacks, putting 49 pounds into each. How many sacks did he fill?

57. In how many days can 56 men do 2,464 days' work?

58. A dealer received $1,625 for sewing-machines, at $65 apiece. How many machines did he sell? *25.*

59. In a certain town the bounties paid to 93 soldiers amounted to $78,771. How much was each soldier's bounty?

60. A hop grower sold 17 bales of hops, which weighed 3,536 pounds. What was the weight of each bale?

61. A farmer harvested 2,520 bushels of oats from 36 acres. What was the yield per acre? *70 bushels.*

62. A fish dealer packed 1,200 pounds of mackerel in kits holding 25 pounds each. How many kits did he fill? *48.*

63. A New York daily newspaper publisher uses 96 reams of paper each day. How many days will 10,080 reams last him?

64. In how many days can 38 men do as much work as one man can do in 1,862 days? *49.*

65. Divide 7,011 by 57. *Quotient, 123.*

66. Divide 126,378 by 63. *Quotient, 2,006.*

67. What is the quotient of 11,952,983 ÷ 569?

68. What is the quotient of 7,000,888 ÷ 758?

69. The dividend is 4,235,262, and the divisor 1,294. What is the quotient? *3,273.*

70. Find the quotient of 20,438,574 ÷ 4,082.

CASE III.

Remainders after Dividing last Partial Dividend.

89. Ex. How many times is 7 contained in 25?

EXPLANATION.—Since 7 is contained in 21, 3 times, and in 28, 4 times; and since 21 is 4 less than 25, and 28 is more than 25; 7 is contained in 25, 3 times, with a remainder of 4.

SOLUTION.

$$\begin{array}{r|l} 25 & 7 \\ 21 & 3 \end{array}$$

4 Remainder.

PROBLEMS.

71. What will be the remainder, when 529 is divided by 34?

SOLUTION OF PROBLEM 71.

$$\begin{array}{r|l} 529 & 34 \\ 34 & 15 \\ \hline 189 & \\ 170 & \\ \hline 19 & \text{Remainder} \end{array}$$

72. How many times is 13 contained in 217? *16 times, with a remainder of 9.*

73. Into how many farms of 156 acres each can 798 acres be divided?

Into 5 farms, with 18 acres left.

74. How many dress patterns of 13 yards each can be cut from a piece of mohair cloth containing 43 yards?

3, with a remnant of 4 yards.

75. What will be the quotient and what the remainder, when 3,376 is divided by 65? *Quotient, 51; remainder, 61.*

76. The dividend is 51,327, and the divisor 807. Find the quotient and the remainder. *Quotient, 63; remainder, 486.*

77. A dairy-man went to an auction with $318, and bought as many cows at $48 each as he could pay for. How many cows did he buy, and how many dollars had he left?

He bought 6 cows, and had $30 left.

78. How many kettles, each weighing 348 pounds, can be made from 20,000 pounds of iron?

57, and 164 pounds of iron will be left.

79. The dividend is 246,875, and the divisor 1,159. What is the quotient, and what the remainder?

Quotient, 213; remainder, 8.

80. The dividend is 705,000, and the divisor 3,275. Find the quotient and the remainder. *Remainder, 875.*

81. How many sewing-machines at $48 each can an agent buy with $1,185? *24 machines, and have $33 left.*

82. A silversmith had 1,096 ounces of silver. After making as many silver pitchers, each weighing 45 ounces, as possible, he made a cake basket of the silver he had left. How much did the cake basket weigh? *16 ounces.*

CASE IV.

The Divisor any Number of Tens, Hundreds, and so on.

90. Ex. 1. Divide 2,750 by 10.

SOLUTION.

$$\begin{array}{r|l} 2750 & 10 \\ 20 & \overline{275} \\ \hline 75 & \\ 70 & \\ \hline 50 & \\ 50 & \\ \hline \end{array}$$

EXPLANATION.—By this Solution it will be seen that the figures of the quotient are the same as those of the dividend after removing the right-hand figure.

Ex. 2.—Divide 32,764 by 100.

EXPLANATION.—By the First Solution we see that if we omit the two right-hand figures of the dividend, the other figures are the same as the quotient; and that the two right-hand figures thus omitted are the same as the remainder. Hence,

We may write all of the dividend except the two right-hand figures for the quotient, and the two right-hand figures for the remainder, as shown in the second solution.

FIRST SOLUTION.

$$\begin{array}{r|l} 32764 & 100 \\ 300 & \overline{327} \\ \hline 276 & \\ 200 & \\ \hline 764 & \\ 700 & \\ \hline 64 & \end{array}$$

SECOND SOLUTION.

$$\begin{array}{r|l} 32764 & 100 \\ \hline 327 & 64 \end{array}$$

91. *We may divide by* 10 *by removing the right-hand figure of the dividend; by* 100 *by removing the two right-hand figures; by* 1,000 *by removing the three right-hand figures; and by any similar number, by removing as many figures from the right of the dividend as there are ciphers in the divisor.*

92. *The figures removed will be the remainder.*

PROBLEMS.

83. At $10 a ton how much hay can I buy for $450? *45 tons.*

84. One day the New York and Brooklyn ferries received 87,600 cents from passengers. How many dollars did they receive, there being 100 cents in one dollar? *$876.*

85. An army consisting of 375,000 soldiers was divided into regiments of 1,000 men each. How many regiments were in the army? *375.*

86. Divide 647,500 by 100. *Quotient, 6,475.*

87. What is the quotient of 1,627,000÷1,000?

88. What is the quotient of 324,700,000÷10,000?

89. The dividend is 725,000,000 and the divisor 100,000. What is the quotient?

90. A farmer having $187, bought yearling cattle at $10 each. How many yearlings did he buy, and how much money had he left? *He bought 18 yearlings, and had $7 left.*

91. How many horses at $100 each can I buy with $2,765? *I can buy 27 horses, and have $65 left.*

92. What is the quotient of 76,275÷1,000? *Quotient, 76; remainder, 275.*

93. The dividend is 32,967,816, and the divisor 10,000. What is the quotient, and what the remainder? *Quotient, 3,296; remainder, 7,816.*

93. Ex.—Divide 53,485 by 700.

FIRST SOLUTION.

53485 | 700
4900 | 76
4485
4200
285

Explanation.—By the First Solution we find the quotient to be 76, and the remainder 285.

700 is 100 times 7. Hence we may first divide by 100, and the quotient thus obtained by 7. Dividing by 100, we have a quotient of 534, with a remainder of 85. Then dividing by 7, we have

SECOND SOLUTION.

53485 | 100 × 7
534 85
Quotient 76 2
285 Remainder.

a quotient of 76, with a remainder of 2. This remainder 2 prefixed to the first remainder 85, makes 285 the whole remainder.

In the Third Solution we first cut off by a vertical line the ciphers from the divisor, and as many figures from the right of the dividend. We then divided the remaining figures of the dividend by the remaining figures of the divisor; and for the remainder, annexed 85, the two figures cut off from the dividend, to 2, the remainder obtained by dividing by 7.

THIRD SOLUTION.

534 | 85 | 7 | 00

76 285

94. *Rules for Division of Integers.*

I. For Long Division.

1. *Place the divisor at the right of the dividend, separate them by a line, and draw a line under the divisor to separate it from the quotient.*

2. *Find how many times the divisor is contained in the first left-hand figure or figures of the dividend, and write the result under the divisor for the first figure of the quotient.*

3. *Multiply the divisor by this quotient figure, and write the product under the figures of the dividend already used.*

4. *Subtract this product from the figures above it, and to the remainder annex the next figure of the dividend, thus forming a partial dividend.*

5. *Find how many times the divisor is contained in this partial dividend, and write the result as the second figure of the quotient.*

6. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend, and to the remainder annex the next figure of the dividend.*

7. *Proceed in the same manner until all the figures of the dividend have been used.*

II. For Short Division.

1. *Write the dividend and divisor as before, and draw a line under the dividend to separate it from the quotient.*

2. *Find how many times the divisor is contained in the first left-hand figure or figures of the dividend, as in long division, and write the result under the last figure of the dividend so used, for the first figure of the quotient.*

3. *Multiply, subtract, and form a partial dividend, as in long division, performing the operations mentally.*

4. *Divide this partial dividend, and write the result as the second figure of the quotient.*

5. *Proceed in the same manner until all the figures of the dividend have been used.*

III. When one or more of the right-hand figures of the divisor are ciphers.

1. *Cut them off by a line, and also an equal number of figures from the right of the dividend.*

2. *Divide the remaining figures of the dividend by the remaining figures of the divisor.*

3. *For the true remainder, annex to the last remainder the figures cut off from the dividend.*

PROBLEMS.

94. Divide 299,392 by 24,000.

SOLUTION OF PROBLEM 94.

299 | 392 | 24 | 000
24 — 12 Quotient.
59
48
11392 Remainder.

95. A miller purchased 9,478 pounds of wheat. How many bushels did he buy, allowing 60 pounds to the bushel? *157 bushels 58 pounds.*

96. In one ream of paper there are 20 quires. How many full reams in 1,976 quires? *98 reams 16 quires.*

97. What is the quotient of 387,695 ÷ 4,500? What is the remainder? *Remainder, 695.*

98. How many dollars must a man save each year, to save $2,456 in 8 years? *$307.*

99. An army of 46,872 men had one tent for every 9 soldiers. How many tents were in the army? *5,208.*

100. What is the quotient of 2,554,772 ÷ 124?

101. What is the quotient of 746,853 ÷ 309?

102. A tinsmith made 1,872 blacking boxes, using 1 sheet of tin for every 6 boxes. How many sheets of tin did he use?

103. A pork buyer packed 237,600 pounds of pork, putting 200 pounds into a barrel. How many barrels did he fill?

104. What is the quotient of 524,006,487 ÷ 9? *58,222,943.*

105. Divide 919,734,140 by 22,705.

106. Divide 503,371,278 by 23,259. *Quotient 21,642.*

107. What is the quotient of 18,382,959 ÷ 56,217?

108. The dividend is 240,900,005 and the divisor 86,005. What is the quotient?

109. A drover paid $7,194 for 218 head of cattle. What was the cost per head?

110. How many church bells, each weighing 3,245 pounds, can be made from 77,880 pounds of bell-metal?

111. An army contractor paid $39,865 for 2,345 barrels of beef. How much did the beef cost him per barrel?

112. If 272,384 pounds of meat are distributed to 9,728 soldiers in a month, how many pounds does each soldier receive? *28.*

113. The dividend is 225,072,740 and the divisor 43,167. What is the remainder? *2.*

114. What will be the quotient and what the remainder, when 60,190,105 is divided by 20,006? *Remainder, 12,057.*

115. If a farmer makes 356 gallons of cider, how many casks can he fill, putting 40 gallons into each?

8 casks, and have 36 gallons left.

116. The dividend is 382,775 and the divisor 2,500. What is the quotient, and what the remainder?

117. The dividend is 87,693,275 and the divisor 41,700. Find the quotient and the remainder.

118. Divide 8,329,659 by 365,000.

Quotient, 22; remainder, 299,659.

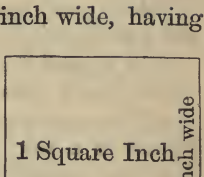
SECTION VI.

MEASUREMENTS.

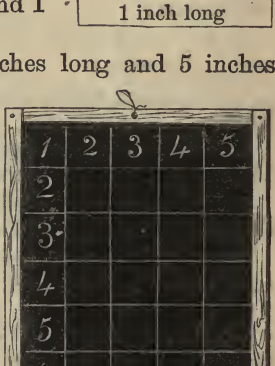
CASE I.

Measurement of Surface.

95. A surface 1 inch long and 1 inch wide, having square corners, is a *Square Inch.* One that is 1 foot long and 1 foot wide is a *Square Foot,* and one that is 1 yard long and 1 yard wide is a *Square Yard.* A *Square Rod* is 1 rod long and 1 rod wide, and a *Square Mile* is 1 mile long and 1 mile wide.



96. If you have a slate 8 inches long and 5 inches wide, and you draw lines just one inch apart from top to bottom, and also from side to side, your slate will be divided into squares each 1 inch long and 1 inch wide. In each of the 5 rows, counting from top to bottom, there are 8 square inches. Since there are 5 rows upon the slate, and in each row there are 8 square inches, in all there are 5 times 8 square inches, or 40 square inches. The number of square inches in 1 row is the same as the number of inches in the length of the slate, and the number of rows is the same as the number of inches in the width of the slate.



PROBLEMS.

1. In a village school there is a blackboard 16 feet long and 5 feet wide. If lines be drawn upon it so as to divide it into square feet, how many square feet will there be in one row? How many rows will there be? How many square feet are there in the blackboard? *80 square feet.*

2. How many square yards are there in a floor 6 yards long and 5 yards wide? *30.*

3. In a pane of glass 22 inches long and 15 inches wide are how many square inches? *330.*

4. How many panes of glass are there in a window that has 6 panes in a row, counting from top to bottom, and 4 panes in a row, counting from side to side?

5. In a village lot 12 rods long and 5 rods wide, how many square rods? *60.*

6. How many square miles in a tract of land 9 miles long and 7 miles wide?

7. How many square feet in a board 16 feet long and 1 foot wide? *16.*

8. How many square yards in the ceiling of a room that is 10 yards long and 7 yards wide?

9. How many square miles in a township 6 miles square, that is, 6 miles long and 6 miles wide? *36.*

10. A boy has a checker-board 12 inches square. How many square inches does it contain? *144.*

97. Ex. A sheet of tin that contains 308 square inches, is 22 inches long. How many inches wide is it?

SOLUTION.

$$\begin{array}{r|l} 308 & 22 \\ 22 & \overline{14} \\ \hline 88 & \\ 88 & \end{array}$$

EXPLANATION.—Since it is 22 inches long, in one row there are 22 square inches; and since in the whole sheet there are 308 square inches, there must be as many rows of square inches as the number of times 22 is contained in 308, which is 14. There are 14 rows; and since each row is one inch wide, the whole sheet is 14 inches wide.

PROBLEMS.

11. If a field is 44 rods long, how wide must it be to contain 1,584 square rods? *36 rods.*

12. A window curtain that is 3 feet wide contains 18 square feet. How long is it? *6 feet.*

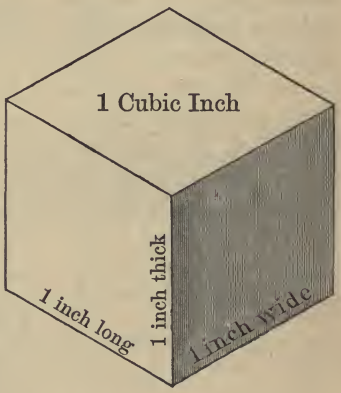
13. A carpenter used 255 square feet of boards in laying the floor of a room 17 feet long. How wide was the room?

14. Mr. White's farm contains 17,280 square rods, and it is 144 rods long. How wide is it? *120 rods.*

CASE II.

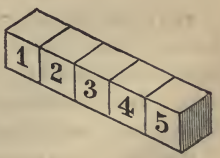
Measurement of Capacity.

98. A block, having square corners and measuring 1 inch each way, or a box, the hollow of which is 1 inch square and 1 inch deep, contains a cubic inch. Any body or portion of space 1 inch long, 1 inch wide, and 1 inch thick is a *Cubic Inch.*



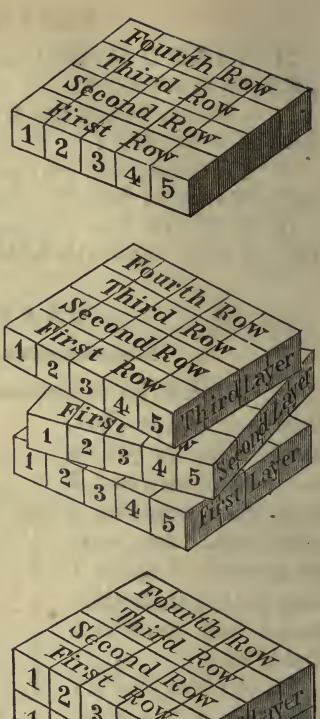
99. A *Cubic Foot* is 1 foot long, 1 foot wide, and 1 foot thick; and a *Cubic Yard* is 1 yard long, 1 yard wide, and 1 yard thick.

100. If 5 blocks, each containing 1 cubic inch, be placed side by side, they will form a row 5 inches long, 1 inch wide, and 1 inch thick. If 4 such rows be placed side by side, they will form a



layer of blocks 5 inches long, 4 inches wide, and 1 inch thick. As there are 4 rows, and in each row five blocks, there are in the layer 4 times 5 blocks, or 20 blocks.

In 3 such layers there are 3 times 20 blocks or 60 blocks. If we place the three layers, one exactly upon the other, they will form a pile 5 inches long, 4 inches wide, and 3 inches high. There are as many cubic inches in one row as there are inches in the length of the pile, as many rows as there are inches in the width of the pile, and as many layers as there are inches in the height of the pile.



PROBLEMS.

15. Henry made a pile of bricks in which there were 4 layers, 5 rows in each layer, and 8 bricks in each row. How many bricks were in the pile? *160.*

16. How many cubic inches in a block 9 inches long, 8 inches wide, and 3 inches thick? *216.*

3 layers, 8 rows in each layer, and 9 cubic inches in each row.

17. A brick is 8 inches long, 4 inches wide, and 2 inches thick. How many cubic inches does it contain? *64.*

18. A farmer has a bin that is 12 feet long, 6 feet wide, and 5 feet deep. How many cubic feet does it contain? *360.*

19. How many cubic yards of earth will be removed in digging a cellar 8 yards long, 6 yards wide, and 2 yards deep?

20. How many cubic feet of stone will it take to build a wall 125 feet long, 6 feet high, and 2 feet thick? *1,500.*

21. In a stick of timber 24 feet long, 1 foot wide, and 1 foot thick, how many cubic feet? *24.*

101. Ex.—A boy who had 120 blocks, made a pile of them, putting 8 blocks in each row, and 3 rows in each layer. How many blocks high was the pile?

EXPLANATION. — Since in 1 row there were 8 blocks, in the 3 rows, or 1 layer, there were 3 times 8 blocks, or 24 blocks. Since in the whole pile there were 120 blocks, and in 1 layer 24 blocks, there were in the pile as many layers as the number of times 24 is contained in 120, which is 5. As there were 5 layers each one block in thickness, the pile was 5 blocks high.

SOLUTION.

$3 \times 8 = 24$

$120 \mid 24$

$120 \quad\quad 5$

PROBLEMS.

22. At another time, with the 120 blocks, the boy made a pile 4 blocks wide, and 3 blocks high. How many blocks were in the length of the pile? *10.*

23. He afterward made a pile 6 blocks long, and 4 blocks high, using all of his blocks. How many blocks wide was the pile? *5.*

24. A block of marble that contains 6,912 cubic inches, is 48 inches long and 18 inches wide. How thick is it?

25. A mason has a pile of stone that contains 585 cubic feet. It is 5 feet high and 9 feet wide. How long is it? *13 feet.*

26. A farmer has in his granary a bin which contains 168 cubic feet. Its length is 7 feet, and its width 6 feet. What is its depth? *4 feet.*

27. In digging a cellar 7 yards long and 2 yards deep, a man removed 70 cubic yards of earth. How wide was the cellar?

102. Length, width, and thickness are called *Dimensions.*

103. A surface has two dimensions, length and width.

104. A body has three dimensions, length, width, and thickness.

105. The extent of any limited surface is its *Area.*

106. The extent of any body or portion of space having three dimensions is its *Capacity.*

107. From the explanations given in Cases I. and II. of this section, we deduce the following

General Principles of Measurement.

I. *The area of any surface having square corners is equal to the product of its two dimensions.*

II. *Either dimension of a surface is equal to the quotient obtained by dividing the area by the other dimension.*

III. *The capacity of any body having square corners is equal to the product of its three dimensions.*

IV. *Any one of the dimensions of such a body is equal to the quotient obtained by dividing the capacity by the product of the other two dimensions.*

PROBLEMS.

28. The floor of a church is 96 feet long and 40 feet wide. How many square feet does it contain? *3,840.*

29. A farmer, measuring his meadow, finds that it is 55 rods long and 32 rods wide. How many square rods does it contain? *1,760.*

30. If 48 square yards of oil-cloth will cover the floor of a dining-room 8 yards long, how wide is the room? *6 yards.*

31. A man used 1,780 square feet of boards in building a tight board fence 5 feet high. What was the length of the fence? *356 feet.*

32. I have a box 11 inches long, 7 inches wide, and 3 inches deep on the inside. How many cubic inches does it contain?

33. A man used 896 cubic feet of stone in building a wall 4 feet high and 2 feet wide. How long was the wall?

34. If 27 cubic feet of earth make one wagon load, how many wagon loads will be removed in digging a cellar 33 feet long, 24 feet wide, and 6 feet deep? *176.*

35. The distance from one village to another is 2,304 rods, and the road is 4 rods wide. How many square rods of land in the road? *9,216.*

36. How many square feet in the roof of a building 32 feet long, the distance from the ridge of the roof to the eaves being 15 feet on each side? *960.*

37. At 10 cents a square foot, how many cents will it cost to build a sidewalk 60 feet long and 5 feet wide? *3,000.*

SECTION VII.

PROBLEMS IN INTEGERS.

1. Of a regiment of 1,047 men 7 were regimental officers, and the rest were divided into 10 companies. How many men were in each company? *104.*

2. How many cords of wood at $6 a cord must be given in exchange for 9 barrels of flour at $8 a barrel? *12.*

3. A vessel, after sailing 8 miles an hour for 46 hours, was driven back 272 miles by a storm. How far was she then from the place of starting? *96 miles.*

4. In a certain church 26 pews rent at $36 each, 22 at $24 each, and 30 at $15 each. For how much do they all rent?

5. A carpenter built me a barn in 7 weeks. I paid him $15 a week for his labor, and the lumber cost me $139. How much did the barn cost me? *$244.*

6. A grocer bought 2 cheeses, one weighing 76 pounds and the other 84 pounds, at 15 cents a pound. How many cents will he gain by selling both at 18 cents a pound? *480.*

7. A farmer bought a lumber wagon for $124, and a market wagon for $140, and paid for them in hay at $12 a ton. How many tons of hay did it take? *22.*

8. A cattle dealer bought 38 head of cattle at $27 each, 92 head at $31 each, and 17 head at $35 each. How much did they all cost him? *$4,473.*

9. A postmaster mailed 857 letters on Monday, 463 on Tuesday, 598 on Wednesday, 325 on Thursday, 218 on Friday, and 649 on Saturday. How many letters did he mail that week? *3,110.*

10. I bought 56 acres of wood-land at $45 an acre. After selling the wood in the tree for $1,978, I sold the land at $20 an acre. Did I gain or lose, and how much? *I gained $578.*

11. A farmer had 2 fields of barley, the first containing 17 acres and the second 28 acres. The first yielded 30 bushels to the acre, and the second 34 bushels. How many bushels of barley did he raise? *1,462.*

12. A man traveled in three successive days 54 miles, 67 miles, and 47 miles. What was the average daily distance traveled?

108. EXPLANATION. — By the *Average*, we mean the number of miles he must have traveled each day, to have traveled the whole distance in the three days, traveling the same distance each day. He traveled the sum of 54, 67, and 47 miles, or 168 miles in 3 days. To have traveled 168 miles in 3 days, traveling the same distance each day, in one day he must have traveled one third of 168 miles or 56 miles. 56 miles was the average daily distance.

SOLUTION.

54
67
47
168 | 3
56

13. A grocer bought three hogsheads of molasses containing respectively 135 gallons, 143 gallons, and 127 gallons. What was the average number of gallons to a hogshead? *135.*

14. In a village school the number of pupils in attendance on Monday was 134, on Tuesday 128, on Wednesday 143, on Thursday 133, and on Friday 147. What was the average daily attendance? *137 pupils.*

15. At a carpet manufactory 19,110 yards of carpet were woven in 78 days. What was the average number of yards woven daily? *245.*

16. A farmer fattened 6 hogs which weighed respectively 312 pounds, 351 pounds, 372 pounds, 395 pounds, 417 pounds, and 451 pounds. What was their average weight? *383 pounds.*

17. A merchant's sales on Monday amounted to $348, on Tuesday to $317, on Wednesday to $294, on Thursday to $336, on Friday to $322, and on Saturday to $369. How much were his average daily sales? *$331.*

18. A farmer bought 28 acres of land at $36 an acre, and 35 acres at $27 an acre. What was the average price per acre?

19. A drover bought 8 cows at $28 each, and 10 cows at $37 each. How much was their average cost? *$33.*

20. How many times can 114 be subtracted from 2,622? *23.*

21. How many times can I subtract 500 from 6,575?
13 times, with a final remainder of 75.

22. A produce dealer sold 28 barrels of pork at $22 a barrel, and expended the money for clover seed at $8 a bushel. How much clover seed did he buy? *77 bushels.*

23. 52 ladies and 39 gentlemen went on an excursion, and their expenses, which were $3 each, were paid by the gentlemen. How much did each gentleman have to pay? *$7.*

24. The two factors of a number are $87 + 48$ and $315 - 142$. What is the number? *23,355.*

25. What is the difference between 67×83 and $59 + 325 + 196$? *5,071.*

26. What is the sum of $31,253 - 8,494$, 127×84 and $6,124 + 3,297$? *42,848.*

27. A hardware merchant bought 18 sets of carriage springs weighing 60 pounds each, 29 sets weighing 37 pounds each, and 64 sets weighing 45 pounds each. How many sets did he buy, and how much did they all weigh?

They all weighed 5,033 pounds.

28. A grocer commenced business with $2,500. The first year he gained $687, and the second year he lost $1,428. The next four years his average yearly gain was $863. How much was he worth at the end of the six years? *$5,211.*

29. If I buy 137 acres of land at $64 an acre, and pay out $876 for fences and buildings upon it, how much does it cost me? *$9,644.*

30. A man buys a farm for $7,865, agreeing to make one payment each year, each payment except the last to be $500. How many payments will he have to make, and what will be the last payment? *16 payments, the last one $365.*

31. A coal dealer bought 350 tons of coal, receiving 2,240 pounds for a ton. He sold it at 2,000 pounds for a ton. How many tons did he sell? *392.*

32. A grocer bought, at different times, 198 pounds of butter, 57 pounds, 324 pounds, 96 pounds, 696 pounds, and 197 pounds. He packed it in tubs, putting 56 pounds into each. How many tubs did he fill? *28.*

33. A man has a lot 60 feet front and 126 feet deep, but as it is low he wishes to fill it in 2 feet. How many wagon loads of earth will be required, 27 cubic feet being one load? *560.*

34. A man paid 3 cents a day for the schooling of each of his 3 children. The children were in school 5 days each week for 42 weeks in the year. How much did their year's schooling cost him? *1,890 cents.*

35. The same man expended for cigars 6 cents a day for each of the 365 days of the year. Which cost him the more, his cigars or the schooling of his children? How much more?

His cigars cost 300 cents more.

36. A young man worked a year at $25 a month. He paid $8 a month for board, and expended $100 more. How much money did he save? *$104.*

37. What is the average weight of 8 bales of cotton which weigh respectively 385, 367, 418, 374, 396, 405, 373, and 402 pounds? *390 pounds.*

38. A manufacturer made 2,468 yards of cloth, and sold 1,382 yards for $4,146, and the remainder for $4,344. How much did he receive per yard for each of the two lots?

$3 a yard for the first, and $4 a yard for the second.

39. A merchant bought 17 pieces of alpaca, each piece containing 42 yards. After selling 140 yards, how many dress patterns, of 14 yards each, had he left? *41.*

40. If I buy a mill for $12,675, and pay down $1,675, how many payments of $1,375 each shall I have to make to pay the balance. *8.*

41. Harry is 6 years old, and his grandmother is 10 times as old as he. Should they both live until he is 9 years old, how many times as old as Harry will his grandmother then be? 7.

42. At a cotton mill 1,071,399 yards of cloth were made in 313 days. How many yards were made daily? *3,423.*

43. In the manufacture of this cloth 357,133 pounds of cotton were used. How many pounds were used daily?

44. How many yards of cloth were made from each pound of cotton? *3.*

45. A farmer had three flocks of sheep, the first containing 184, the second 218, and the third 65 sheep. He sold 114 sheep from the first flock, 189 from the second, and 48 from the third. How many sheep did he sell, and how many had he left? *He had 116 sheep left.*

46. A railroad embankment is 1,300 yards long; its average width is 28 yards, and its average height 10 yards. How many cubic yards of earth does it contain? *364,000.*

47. If 2 bushels of onions can be raised from 1 square rod of ground, how many bushels can be raised in a garden 12 rods long and 9 rods wide? *216.*

48. What is the quotient of 14 times 396 divided by 33 times 42? *4.*

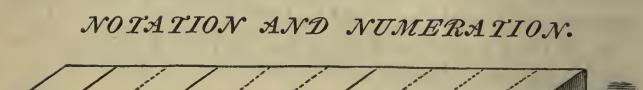
49. From 2,738 + 5,293 + 137,296 subtract 3,279, and divide the remainder by 193. *Quotient 736.*

CHAPTER II.

DECIMALS.

SECTION I.

NOTATION AND NUMERATION.



109. If we divide a block into 10 equal parts, 1 of the parts is 1 tenth of the whole block, 2 of the parts are 2 tenths, 3 of the parts are 3 tenths, and 4 of the parts are 4 tenths. In the whole block there are 10 tenths.

When any thing or number is divided into 10 equal parts, 1 of the parts is *1 tenth* of the thing or number, 3 of the parts are *3 tenths*, 5 of the parts *5 tenths*, 6 of the parts *6 tenths*, 7 of the parts *7 tenths*, and so on.

1 tenth is written .1

2	tenths	are	written	.2	6	tenths	are	written	.6
3	"	"	"	.3	7	"	"	"	.7
4	"	"	"	.4	8	"	"	"	.8
5	"	"	"	.5	9	"	"	"	.9

110. The number 111 consists of 1 hundred 1 ten and 1 one. Since in 1 hundred there are 10 tens, the 1 ten is 1 tenth of 1 hundred; and since in 1 ten there are 10 ones, the 1 one is 1 tenth of 1 ten. Hence,

The value of any figure in a number is 1 tenth of the value of a like figure standing in the next place at the left.

111. The value of a figure written at the right of ones is 1 tenth as great as the value of a like figure in the ones' place, and therefore it is *tenths*.

11 and 1 tenth is written *11.1*
5 and 3 tenths " " *5.3*
16 and 5 tenths " " *16.5*
510 and 6 tenths " " *510.6*

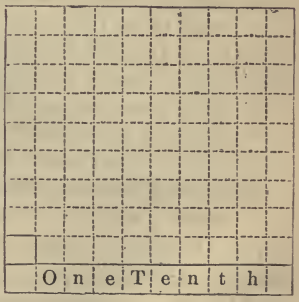
112. The period or point which is placed before tenths is the *Decimal Point*. The figure at the left of the decimal point is always *ones*, and the figure at the right of it is always *tenths*.

When the decimal point stands between figures, it is read *and*. Thus, 5.2 is read 5 and 2 tenths; 290.4 is read 290 and 4 tenths.

EXERCISES.

1. Read .3, .9, .5; 5.2, 18.1, 40.6.
2. Write 2 and 5 tenths.
3. Write 6 and 4 tenths.
4. Write 8 tenths.
5. Write 124 and 7 tenths.
6. Write 50 and 5 tenths.
7. Write 32700 and 1 tenth.
8. Read 7.1, 65.8, 393.3, 200.9, 2363.4.

113. If a board 10 inches square be divided into strips 1 inch wide, each strip will be 1 tenth of the whole board. If each of these strips be divided into 10 equal parts, in the 10 strips, or the whole board, there will be 10 times 10 or 100 equal parts. 1 of these parts will therefore be 1 hundredth of the board.



When 1 tenth of any thing or number is divided into 10 equal parts, each part will be 1 *hundreth* of the whole thing or number.

114. Since the value of a figure in any place is 1 tenth of the value of a like figure in the next place at the left, a figure written at the right of tenths must be *hundreths.* Thus,

.11 is one tenth and 1 hundredth.
.83 is 8 tenths and 3 hundredths.
3.27 is 3 ones, 2 tenths, and 7 hundredths.
.05 is 0 tenths and 5 hundredths.

115. .35 is 3 tenths and 5 hundredths ; but 3 tenths = 30 hundredths, and 30 hundredths + 5 hundredths = 35 hundredths.

Tenths and hundredths are read together as hundredths.

.52 is 5 tenths and 2 hundredths, and is read 52 hundredths.
.96 is 96 hundredths.
.03 is 3 hundredths.
5.24 is 5 and 24 hundredths.
298.05 is 298 and 5 hundredths.

EXERCISES.

9. Read 11.18, 10.24, 81.6.
10. Read 40.93, 128.52, 50.07.
11. Read 7.08, 217.01, 3000.02.
12. Write 7 tenths and 3 hundredths, or 73 hundredths.
13. Write 5 tenths and 1 hundredth, or 51 hundredths.
14. Write 27 hundredths; 3 hundredths.
15. Write 4 and 15 hundredths; 4 and 5 hundredths.
16. Write 800 and 21 hundredths.
17. Write 18000 and 1 hundredth.

116. A figure at the right of hundredths is *thousandths ;* and tenths, hundredths, and thousandths are read together as thousandths.

.456 is 4 tenths, 5 hundredths, and 6 thousandths, and is read 456 thousandths.

.209 is 2 tenths, 0 hundredths, and 9 thousandths, and is read 209 thousandths.

.063 is read 63 thousandths.

.004 is read 4 thousandths.

3.528 is read 3 and 528 thousandths.

80.082 is read 80 and 82 thousandths.

117. Numbers expressed by ones, tens, hundreds, etc., are *Integers* or *Whole Numbers.*

118. Numbers expressed by tenths, hundredths, thousandths, etc., are *Decimals.*

119. A number consisting of an integer and a decimal is a *Mixed Number.*

120. In writing decimals and mixed numbers the decimal point must always be used.

EXERCISES.

18. Read .275, 7.463, 32.416.
19. Read 86285.419, .507, 700.256.
20. Read 11.092, .048, .002.
21. Read 214.005, .001, 217.908.
22. Write 5 and 376 thousandths.
23. Write 3250 and 615 thousandths.
24. Write 43 thousandths; 81 thousandths.
25. Write 87 and 87 thousandths.
26. Write 401 thousandths; 7 thousandths.
27. Write 9000 and 9 thousandths.
28. Write 18 and 305 thousandths.
29. Write 101000 and 101 thousandths.

121. A figure at the right of thousandths is *ten-thousandths;* and a decimal containing tenths, hundredths, thousandths, and ten-thousandths is read as ten-thousandths.

.2574 is 2 tenths, 5 hundredths, 7 thousandths, and 4 ten-thousandths, and is read 2574 ten-thousandths.

.0452 is 452 ten-thousandths.
6.0048 is 6 and 48 ten-thousandths.
59.0006 is 59 and 6 ten-thousandths.
6000.3001 is 6000 and 3001 ten-thousandths.
204.0809 is 204 and 809 ten-thousandths.

122. A figure at the right of ten-thousandths is *hundred-thousandths.* Thus .32516 is 3 tenths, 2 hundredths, 5 thousandths, 1 ten-thousandth, and 6 hundred-thousandths, and is read 32516 hundred-thousandths.

A figure at the right of hundred-thousandths is *millionths.* Thus .259361 is 259361 millionths.

A figure at the right of millionths is *ten-millionths,* and a figure at the right of ten-millionths is *hundred-millionths.*

.63015 is 63015 hundred-thousandths.
.40003 is 40003 hundred-thousandths.
.029304 is 29304 millionths.
.000007 is 7 millionths.
.2367592 is 2367592 ten-millionths.
.0000024 is 24 ten-millionths.
.59642108 is 59642108 hundred-millionths.
.00000009 is 9 hundred-millionths.

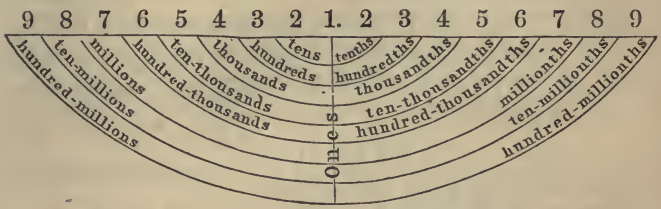
123. TABLE OF VALUES OF DECIMAL NUMBERS.

One	decimal figure	expresses		*tenths.*
Two	decimal figures	express		*hundredths.*
Three	"	"	"	*thousandths.*
Four	"	"	"	*ten-thousandths.*
Five	"	"	"	*hundred-thousandths.*
Six	"	"	"	*millionths.*
Seven	"	"	"	*ten-millionths.*
Eight	"	"	"	*hundred-millionths.*

(See Manual, page 218.)

124. Figures standing in places at equal distances to the right and left of ones have *names* that correspond to each other, as shown in the following

DIAGRAM OF DECIMAL NOTATION.



125. *The place which any decimal figure occupies in a number determines the value expressed by it in that number.*

126. The relative values of the different places in decimals are shown by the following

DECIMAL NOTATION AND NUMERATION TABLE.

1 one	is	10 tenths.
1 tenth	"	10 hundredths.
1 hundredth	"	10 thousandths.
1 thousandth	"	10 ten-thousandths.
1 ten-thousandth	"	10 hundred-thousandths.
1 hundred-thousandth	"	10 millionths.
1 millionth	"	10 ten-millionths.
1 ten-millionth	"	10 hundred-millionths.
10 tenths	are	1 one.
10 hundredths	"	1 tenth.
10 thousandths	"	1 hundredth.
10 ten-thousandths	"	1 thousandth.
10 hundred-thousandths	"	1 ten-thousandth.
10 millionths	"	1 hundred-thousandth.
10 ten-millionths	"	1 millionth.
10 hundred-millionths	"	1 ten-millionth.

(See Manual, page 218.)

127. Since the value of every figure in a decimal is determined by the place it occupies, and since ciphers on the right of a decimal do not change the places of the other figures in the number, it follows that

I. *Ciphers may be annexed to any decimal, or decimal ciphers to any integer, without changing its value; and*

II. *Ciphers may be omitted from the right of any decimal, or decimal ciphers from the right of any integer, without changing its value.* (See Manual, page 218.)

EXERCISES.

(See Manual, page 218.)

30. Read .523, 4.276, .009, .027, .209.

31. Read 9.018, 100.003, 435.125.

32. Read .2987, 4.0232, 18.0901, .1805, 14.0029.

33. Read 365.0007, 60.1273, 400.5017.

34. Read .7025, .7005, .0005, 30.6008.

35. Read .24731, .09671, .10006, .00008.

36. Read .00055, .70438, 8.52804.

37. Read .90052, 250.46031, 349.30116, 1000.20084.

38. Read .256153, 709.400365, .4366576.

39. Read .1413948, 32876.2850041, 25.530016.

40. Read 217.1800624, 59.00654387, 3054.26405746.

41. Read 32957251.5283563, 172000650.50040036.

42. Write sixteen ten-thousandths.

43. Write five hundred seventeen and three thousand six hundred forty-seven ten-thousandths.

44. Write thirty-six thousand two hundred seventy-three hundred-thousandths.

45. Write fourteen hundred-thousandths.

46. Write five thousand eighteen hundred-thousandths.

47. Write two hundred seventeen thousand four hundred fifty-six millionths.

48. Write six hundred thousand two hundred eighty-four millionths.

49. Write one hundred ninety-three millionths.

50. Write sixteen million three hundred fifty-eight thousand seven hundred twenty-four ten-millionths.

51. Write forty-six million two hundred seventy-four thousand five hundred eight hundred-millionths.

52. Write 78,219 ten-millionths.

53. Write 6 and 49 hundred-millionths.

54. Write 106,204 hundred-millionths.

55. Write 7,017 and 4 millionths.

56. Write 4 and 68,001 ten-millionths.

57. Write 44 and 44 hundred-thousandths.

58. Write 975 million 206 thousand 410 and 3 tenths.

59. Write 50 million and 50,512 hundred-millionths.

60. Write 101 million 101 thousand 101 and 1,001,001 hundred-millionths.

SECTION II.

ADDITION.

128. Ex. What is the sum of 45.75, 29.36, and 4.42?

EXPLANATION.—We write tens under tens, ones under ones, tenths under tenths, and hundredths under hundredths. The decimal points then stand in a column. We commence at the right, and add as in integers. The sum of the column of hundredths is hundredths, and the sum of the column of tenths is tenths. We must therefore place the decimal point at the left of the 5 in the sum. (See Manual, page 219.)

SOLUTION.

45.75
29.36
4.42
79.53

129. *The decimal point in the sum must always be placed directly below the decimal points in the parts.*

PROBLEMS.

1. A farmer had 98.7 acres of land, and bought 15.5 acres more. How many acres had he then? *114.2.*

2. A lady bought two carpets, the first containing 27.5 yards, and the second 23.5 yards. How many yards of carpeting did she buy? *51 yards.*

3. A farmer who had three meadows cut from the first 18.68 tons of hay, from the second 15.27 tons, and from the third 13.54 tons. How much hay did he cut from the three meadows?

4. A silversmith used 3.65 ounces of silver in making a set of tea-spoons, 8.72 ounces for a set of table-spoons, and 11.63 ounces for a set of forks. How much silver did he use for all? *24 ounces.*

5. A gentleman has three village lots, one of which contains .8 of an acre, another .5, and the third 3.4 acres. How many acres has he in the three lots?

6. A housekeeper burned .728 of a ton of coal in December, .835 of a ton in January, .694 of a ton in February, and .532 of a ton in March. How many tons did she burn in the four months? *2.789.*

7. At a Pennsylvania iron mine 216.845 tons of iron were mined on Monday, 204.376 tons on Tuesday, 198.275 tons on Wednesday, 220.615 tons on Thursday, 187.945 tons on Friday, and 206.004 tons on Saturday. How many tons were mined in the week? *1234.06.*

8. A wood dealer has on hand 57.75 cords of hickory wood, 139.75 cords of maple wood, 67.65 cords of beech wood, and 78.26 cords of hemlock wood. How many cords of wood has he? *343.41.*

9. A farmer has four meadows. The first contains 10.15 acres, the second 9.76 acres, the third 12.25 acres, and the fourth 7.82 acres. How much land has he in the four meadows? *39.98 acres.*

10. The cargo of a coal barge consisted of 45.75 tons of chestnut coal, 69.54 tons of stove coal, 36.94 tons of egg coal, and 51.25 tons of lump coal. What was the whole amount of coal in the cargo? *203.48 tons.*

11. A man traveled 125.75 miles by stage, 213.75 miles by railroad, and 89.45 miles by steamboat. How many miles did he travel in all? *428.95.*

13. A tavern keeper bought four loads of hay, the first containing 1.156 tons, the second 1.228 tons, the third .987 tons, and the fourth 1.048 tons. How much hay did he buy?

13. What is the sum of 1.225 + .865 + .655? *2.745.*

130. Ex. What is the sum of 7.4675, 836.5, 85.275, and 973?

EXPLANATION.—After writing the numbers with the decimal points in a column, thus bringing ones under ones, tenths under tenths, etc., we annex ciphers to the second, third and fourth numbers until each contains as many decimal figures as the first number. (See **127.**) We then add as in integers, and place the decimal point in the sum directly under the decimal points in the parts.

SOLUTION.

7.4675
836.5000
85.2750
973.0000
1902.2425

PROBLEMS.

14. A tailor bought two pieces of cassimere, one containing 38.5 yards, and the other 41.25 yards. How many yards of cloth in both pieces? *79.75.*

15. One hour a railroad train ran 22.6 miles, the next hour 28.82 miles, and the third hour 23.27 miles. How far did it run in the three hours? *74.69 miles.*

16. In four successive weeks an ice dealer sold 18.263 tons, 15.967 tons, 17.4 tons, and 16.48 tons of ice. How much ice did he sell in the four weeks? *68.11 tons.*

17. How many tons of straw are there in five stacks, the first of which contains 3.7 tons, the second 4.1275 tons, the third 2.875 tons, the fourth 5.2 tons, and the fifth 7.65 tons? *23.5525.*

18. A fruit dealer bought five lots of cranberries. The first lot contained 5.75 bushels, the second lot 8.5 bushels, the third lot 6.625 bushels, the fourth lot 9 bushels, and the fifth lot 4.25 bushels. How many bushels were there in the five lots?

131. *Rule for Addition of Decimals.*

I. *Write the numbers so that the decimal points shall stand in a column.*

II. *Add in the same manner as in integers, and place the decimal point in the sum directly under the decimal points in the parts.*

PROBLEMS.

19. What is the sum of 5.0084 + 641.385 + 9.00843 + 21.000001 + 5.064 ? *681.465831.*

20. A merchant sold 1.75 bushels of clover seed to one farmer, 3 bushels to another, 2.5 bushels to a third, 4.225 bushels to a fourth, and 3.25 bushels to a fifth. How many bushels did he sell to the five farmers ? *14.725.*

21. What is the sum of five and five tenths, five and five hundredths, eight and seventy-five thousandths, twenty-one and three thousand six ten-thousandths, and five thousand nine and six hundred forty thousand seventeen millionths ? *5049.565617.*

22. A gardener sold 4.5 bushels of beans to one grocer, 7 bushels to another, 3.25 bushels to a third, 1.625 bushels to a fourth, and 2.125 bushels to a fifth. How many bushels did he sell to all ? *18.5.*

23. What is the sum of three hundred and three hundredths, one thousand seven and two hundred thousand six millionths, one hundred seventeen thousand seven hundred nine and six hundred four ten-thousandths, and eight and fifty-two millionths ? *119024.290458.*

(24)	(25)	(26)	(27)
47.25	.967	.125	8000.1
5.00695	.00054	1.25	96.2006
193.9	953.5	12.5	504.40307
5.876	7.375	.0125	2046.25
94.376964	1000.0001	.00125	9.0004
9.00005	6.75	7.3	28.4
290.050063	8.80808	.0827	167.283
	483	.000006	5.5008

SECTION III.

SUBTRACTION.

132. Ex. From 38.25 subtract 16.78.

EXPLANATION.—We write the numbers so that the decimal point of the subtrahend shall stand directly under that of the minuend, and subtract as in integers. Since hundredths subtracted from hundredths leave hundredths, and tenths subtracted from tenths leave tenths, there are hundreths and tenths in the remainder; we therefore place the decimal point before the 4.

SOLUTION.

38.25
16.78
21.47

133. *The decimal point in the remainder must always be directly under the decimal point in the minuend and subtrahend.*

PROBLEMS.

1. A merchant sold 16.7 yards of calico from a piece that contained 33.4 yards. How many yards remained in the piece? *16.7.*

2. A farmer raised 32.25 bushels of clover seed, and sold 24.75 bushels. How many bushels had he left? *7.5.*

3. A druggist bought a cask of wine containing 36.5 gallons. After selling 19.5 gallons, how much remained in the cask? *17 gallons.*

4. At night the ice on the river was 7.37 inches thick, and in the morning it was 9.03 inches thick. How much ice had formed during the night? *1.66 inches thick.*

5. A wood dealer bought 398.65 cords of wood, and sold 276.25 cords. How many cords remained unsold? *122.4.*

6. Of a railroad, which is to be 156.325 miles long when finished, 83.875 miles are built. How many miles remain unfinished? *72.45.*

7. Mr. West's farm is 116.36 rods in length and 29.28 rods less in width. What is its width? *87.08 rods.*

134. Ex. From 130.5 subtract 93.1875.

EXPLANATION.—Since ciphers may be annexed to any decimal number without changing its value, (see **127**), we annex ciphers to the minuend until it contains as many decimal figures as the subtrahend, and then subtract, and place the decimal point as before explained.

SOLUTION.

130.5000
93.1875
37.3125

PROBLEMS.

8. From a cask containing 31.5 gallons of vinegar 20.75 gallons were drawn. How many gallons remained in the cask? *10.75.*

9. A man put into his wood-house 24.5 cords of wood for his year's supply, and at the end of the year he had 2.875 cords left. How many cords had he used? *21.625.*

10. Two men built 134 rods of stone fence, one of them building 65.87 rods. How many rods did the other build?

11. From a hogshead of molasses containing 135.5 gallons, 1.175 gallons leaked out. How many gallons were then in the hogshead? *134.325.*

12. A farmer having 217.625 acres of land, sold 87.0375 acres. How much land had he left? *130.5875 acres.*

135. *Rule for Subtraction of Decimals.*

I. *Write the numbers with the decimal point of the subtrahend directly under that of the minuend.*

II. *Subtract in the same manner as in integers, and place the decimal point in the remainder directly under the decimal point in the subtrahend.*

PROBLEMS.

13. From a box that contains 10 pounds of indigo a grocer sold 7.0625 pounds. How many pounds remained in the box? *2.9375.*

14. From ninety-five and forty-four thousandths take ten and eight thousand five ten-thousandths.

15. A liberty pole 83.5 feet long was set in the ground 8.75 feet. How many feet from the ground to the top of the pole? *74.75*

16. A cubic inch of silver weighs 6.061 ounces, and a cubic inch of marble 1.641 ounces. How much heavier is the silver than the marble? *4.42 ounces.*

17. 923.85 miles — 385.275 miles = how many miles?

18. A silver dollar which weighs 412.5 grains, contains 41.25 grains of copper. How much pure silver does it contain? *371.25 grains.*

19. A lady used 19 yards of silk in making dresses for her two daughters, using 9.875 yards for one dress. How much silk did the other dress contain? *9.125 yards.*

20. 48.2175 acres — 39.5 acres are how many acres?

21. At a certain point near Sandy Hook, N. J., the water at low tide is 5.649 feet deep, and at high tide it is 11.249 feet deep. How much does the tide rise and fall at that point?

22. The walls of a school room measure 120 square yards, but the openings (windows and doors) measure 17.75 square yards. How many yards for plastering are there on the walls?

23. The ceiling of the same room measures 51.5 square yards. How many yards less of plastering in the ceiling than in the walls? *50.75.*

(24)	(25)	(26)
87.006	1.03045	20000.2875
9.84	.0009	482.52006

27. From eight hundred sixty and four hundredths take nineteen and nine thousand eighty-four hundred-thousandths. *Remainder, 840.94916.*

28. A cubic foot of gold weighs 1203.625 pounds, and a cubic foot of iron 450.4375 pounds. How much more does the gold weigh than the iron? *753.1875 pounds.*

29. A vessel sailed from Boston for Havana with a cargo of 438.275 tons of ice, but 156.895 tons melted on the voyage. How much of the ice reached Havana? *281.38*

30. How much more ice reached Havana than melted on the voyage?

SECTION IV.

MULTIPLICATION.

CASE I.

One Factor an Integer.

136. 3 times 3 apples are 9 apples, 3 times 3 ones are 9 ones, and 3 times 3 tens are 9 tens. So, also, 3 times 3 tenths are 9 tenths, 3 times 3 hundredths are 9 hundredths, and 3 times 3 thousandths are 9 thousandths. (See **52**, V.)

.3	.03	.003
3	3	3
.9	.09	.009

Ex. 1. Multiply 43.21 by 4.

EXPLANATION.—We write the factors and multiply as in integers. Since 4 times 1 hundredth are 4 hundredths, and 4 times 2 tenths are 8 tenths, the 4 in the product is hundredths, and the 8 is tenths. We must therefore place the decimal point before the 8.

SOLUTION.

43.21
4
172.84

Ex. 2. Multiply .12815 by 7.

EXPLANATION. — Since the multiplicand is hundred-thousandths, 7 times this multiplicand must also be hundred-thousandths; and since hundred-thousandths are expressed by five decimal figures, (see **123**), there must be five decimal figures in this product. Hence,

SOLUTION.

.12815
7
.89705

When the multiplier is an integer, there are as many decimal figures in the product as in the multiplicand.

PROBLEMS.

1. In one rod there are 16.5 feet. How many feet across a street that is 3 rods wide? *49.5.*

2. How many acres in 8 fields, each containing 12.84 acres?

3. How many bushels of oats are there in 4 bins, each containing 20.75 bushels?

4. How many miles will a man travel in 8 days, if he travels 44.625 miles each day? *357.*

5. If .085 of a pound of butter be made from one quart of milk, how much butter can be made from 9 quarts? *.765 of a pound.*

SOLUTION OF PROBLEM 3.

$$\begin{array}{r} 20.75 \\ 4 \\ \hline 83.00 \end{array}$$

6. A farmer put 8 loads of hay in a stack, and each load weighed .9375 of a ton. How many tons of hay in the stack?

(7)	(8)	(9)	(10)
210.735	634.04	19.125	250.375
9	6	8	6

11. What is the product of 16.24 multiplied by 14?

12. If a man builds 2.5 rods of stone-wall in one day, how many rods can he build in 18 days? *45.*

13. If a mason can plaster 145.75 square yards of wall in a day, how many yards can he plaster in 26 days? *3789.5.*

SOLUTION OF PROBLEM 11.

$$\begin{array}{r} 16.24 \\ 14 \\ \hline 6496 \\ 1624 \\ \hline 227.36 \end{array}$$

14. A merchant bought 39 pieces of sheeting, each piece containing 40.25 yards. How many yards did he buy?

15. What is the weight of 47 reams of printing-paper, each ream weighing 38.125 pounds? *1791.875 pounds.*

16. How many rods of ditch will a laborer dig in 64 days, if he digs 7.38 rods each day? *472.32.*

17. A jeweler made 85 finger rings, using .0225 of an ounce of gold for each ring. How much gold did he use? *1.9125 ounces.*

18. A farmer sheared 113 sheep, and the fleeces averaged 4.0625 pounds. What was the amount of his wool clip? *459.0625 pounds.*

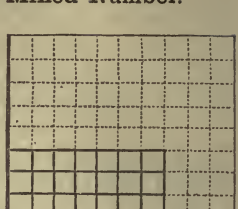
19. If a gardener can raise 405 bushels of onions on one acre of ground, how many bushels can he raise on 3.276 acres?

20. How much is 7 million times 7 millionths? *49.*

21. If 94.2 tons of iron are required for one mile of railroad, how many tons will be required for a road 164.25 miles long?

CASE II.

Each Factor a Decimal or a Mixed Number.



137. If from a board 1 foot square a part be taken, 7 tenths of a foot long and 5 tenths of a foot wide, the area of the part may be found by multiplying its length by its breadth. By the diagram it will be seen that the square foot contains 10 times 10, or 100 small squares ; and hence, each of these small squares is 1 hundredth of the whole board. The part taken contains 5 times 7, or 35 of these small squares, or 35 hundredths of the whole board. Hence, 7 tenths multiplied by 5 tenths must produce 35 hundredths.

.7
.5
.35

138. *The product of tenths multiplied by tenths is hundredths.*

Ex. Multiply 24.3 by .3.

Explanation.—We first multiply as in integers. Then, since the product of tenths multiplied by tenths is hundredths, and since hundredths are expressed by two decimal figures, we place the decimal point before the 2 in the product.

SOLUTION.

24.3
.3
7.29

PROBLEMS.

22. Multiply .8 by .3. *Product, .24.*

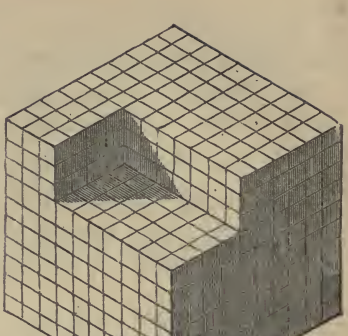
23. What is the product of .9 multiplied by .7 ?

24. What is the product of 16.7 multiplied by .5 ? *8.35.*

25. Multiply 38.3 by 8. *Product, 30.64.*

26. If a man can chop 2.5 cords of wood in a day, how many cords can he chop in .7 of a day ? *1.75.*

27. How many square feet are there in a board 16.5 feet long and .9 of a foot wide ? *14.85.*



139. If from a cubic foot a part be taken, 7 tenths of a foot long, 4 tenths of a foot wide, and 3 tenths of a foot thick, the capacity of the part may be found by multiplying its length, width, and thickness together. By the first figure it will be seen that the whole cubic foot contains 10 times 10 times 10, or 1,000 small cubes; and hence each small cube is 1 thousandth of the cubic foot. By the second figure it will be seen that the part taken contains 7 times 4 times 3, or 84 small cubes, which are 84 thousandths of the cubic foot. Hence, the product of 7 tenths, 4 tenths, and 3 tenths is 84 thousandths.

SOLUTION.

To obtain the product of three factors, we multiply the product of the first two by the third. The product of 7 tenths and 4 tenths is 28 hundredths. Then, 28 hundredths multiplied by 3 tenths must produce 84 thousandths.

$$\begin{array}{r} .7 \\ .4 \\ \hline .28 \\ .3 \\ \hline .084 \end{array}$$

140. *The product of hundredths multiplied by tenths is thousandths.*

When tenths are multiplied by tenths, there is 1 decimal figure in each factor and 2 in the product. When hundredths are multiplied by tenths, there are 2 decimal figures in one factor, 1 in the other, and 3

in the product. In each case there are as many decimal figures in the product as in both factors.

141. *The product must always contain as many decimal figures as both factors.*

Ex. Multiply 44.76 by 8.23.

EXPLANATION.—Since there are four decimal figures in both factors, there must be four decimal figures in the product.

SOLUTION.

44.76
8.23
13428
8952
35808
368.3748

142. *Rule for Multiplication of Decimals.*

I. *Write the numbers, and multiply as in integers.*

II. *Place the decimal point in the product so that it shall contain as many decimal figures as both factors.*

PROBLEMS.

28. What is the product of .43 multiplied by .4? *.172.*

29. Multiply .84 by .6. *Product, .504.*

30. If one ton of iron ore yields .685 of a ton of iron, how many tons of iron will 893.056 tons of ore yield? *611.74336.*

31. How many tons of broom-corn can be raised from .85 of an acre of land, if 1.8764 tons can be raised from one acre? *1.59494.*

32. How many gallons of linseed-oil can be obtained from 249.5 bushels of flaxseed, if 3.15 gallons of oil can be obtained from one bushel of seed? *785.925.*

33. If 3.75 gallons of cider can be made from one bushel of apples, how much cider can be made from 38.5 bushels? *144.375 gallons.*

34. If one yard of cassimere can be made from 1.625 pounds of wool, how many pounds of wool will be required for 54.25 yards? *88.15625.*

35. In one square rod there are 272.25 square feet. How many square feet in 108.4 square rods? *29511.9.*

36. If 22.73 pounds of starch be made from one bushel of corn, how many pounds can be made from 83.25 bushels?

37. Multiply .0854 by 0.32.

Since there are seven decimal figures in both factors, and in the product but five, we must prefix two ciphers to the product, and place the decimal point before the first one.

SOLUTION OF PROBLEM 36.

.0854
.032
1708
2562
.0027328

38. Multiply .084 by .07. *Product, .00588.*

39. What is the product of .00393 multiplied by .006? *.00002358.*

40. What is the product of .057 and .00049? *.00002793.*

41. Multiply .06052 by .066. *Product, .00399432.*

SECTION V.

DIVISION.

CASE I

The Divisor an Integer.

143. One fourth of 8 apples is 2 apples, one fourth of 8 feet is 2 feet, one fourth of 8 ones is 2 ones, and one fourth of 8 tens is is 2 tens. So also one fourth of 8 tenths is 2 tenths, one fourth of 8 hundredths is 2 hundredths, one fourth of 8 thousandths is 2 thousandths, etc.

.8 ⌊ 4	.08 ⌊ 4	.008 ⌊ 4
.2	.02	.002

Ex. 1. Divide .9275 by 7.

EXPLANATION.—We write the numbers and divide as in integers. Since one seventh of 9 tenths is 1 tenth with a remainder, the first quotient figure 1 is tenths, and the decimal point must therefore be placed before it.

SOLUTION.

.9275 ⌊ 7
.1325

Ex. 2. Divide 1047.15 by 45.

EXPLANATION. — Dividing 1047 by 45 we obtain a quotient of 23 and a remainder of 12, as shown in the Partial Solution. Since the three of the quotient is ones, we must place the decimal point after it, before writing the next quotient figure. We then continue the division until all the figures of the dividend have been used.

PARTIAL SOLUTION.

1047.15	45
90	23
147	
135	
12	

SOLUTION.

1047.15	45
90	23.27
147	
135	
121	
90	
315	
315	

144. *When a decimal or a mixed number is divided by an integer, there will be as many decimal figures in the quotient as in the dividend.*

PROBLEMS.

1. A physician fed .675 of a ton of hay to his horse in 5 weeks. How much hay did he feed each week? *.135 of a ton.*

2. If .804 of a ton of coal will last a family 6 weeks, how much do they burn in a week? *.134 of a ton.*

3. A bell-founder cast 8 bells of equal size, and they weighed together .984 of a ton. What was the weight of each bell? *.123 of a ton.*

4. A lady put 16.24 pounds of grapes into 8 fruit cans. How many pounds did she put into each can? *2.03.*

5. A father divided 217.5 acres of land equally among his 5 sons. How many acres did each son receive? *43.5.*

6. If 2.1875 barrels of flour will last a family 7 months, how much flour will they use in one month? *.3125 of barrel.*

145. Ex. 1. Divide 4 by 8.

EXPLANATION.—Since 8 is not contained in 4, we annex a decimal cipher to the four ones before dividing. (See **127.**)

SOLUTION.

4.0	8
.5	

Ex. 2. How many times is 32 contained in 164?

EXPLANATION.—The quotient of 164 divided by 32 is 5 with a remainder of 4, and since the quotient figure 5 is ones, we place the decimal point at the right of it. We then annex decimal ciphers to the remainder 4 ones (see **127**), and continue the division until there is no remainder.

SOLUTION.

$$\begin{array}{r|l} 164 & 32 \\ 160 & \overline{5.125} \\ \hline 4000 & \\ 32 & \\ \hline 80 & \\ 64 & \\ \hline 160 & \\ 160 & \\ \hline \end{array}$$

PROBLEMS.

7. A silver-ware manufacturer made 25 sets of table-spoons which weighed 231 ounces. How much did one set weigh? *9.24 ounces.*

8. An Iowa farmer raised 3045 bushels of corn from 56 acres. How much was the yield to the acre? *54.375 bushels.*

9. If a man can chop 44.625 cords of wood in 21 days, how many cords can he chop in one day? *2.125.*

146. Ex. What is the quotient of .099 divided by 36?

EXPLANATION.—Since one thirty-sixth of 99 thousandths is 2 thousandths with a remainder, we write the 2 in the quotient as thousandths by prefixing two decimal ciphers, and then continue the division until there is no remainder.

SOLUTION.

$$\begin{array}{r|l} .099 & 36 \\ 72 & \overline{.00275} \\ \hline 270 & \\ 252 & \\ \hline 180 & \\ 180 & \\ \hline \end{array}$$

PROBLEMS.

10. The dividend is .897 and the divisor 39. What is the quotient? *.023.*

11. If .7505 of an ounce of gold-leaf will cover 79 square feet, how much gold-leaf will be required to gild one square foot? *.0095 of an ounce.*

12. A manufacturer put up 23 gallons of lemon extract in 736 bottles. How much did each bottle contain?

13. A dairyman made 7.2 pounds of butter from 128 quarts of milk. How much butter was that from one quart of milk?

14. Divide .7 by 112. *Quotient, .00625.*

15. If one bushel of Onondaga salt can be made from 35 gallons of brine, how many bushels can be made from 618.625 gallons? *17.675.*

CASE II.

The Divisor a Decimal or a Mixed Number.

147. The quotient of 15 divided by 5 is 3. If we multiply both dividend and divisor by 4, and divide the new dividend by the new divisor, the quotient will be 3, the same as before. If we multiply both terms by 23 and divide, we obtain the same quotient.

15 ⌊ 5 Divisor.
3 Quotient.

15 Dividend.
23
45 5 Divisor.
30 23
345 | 115 New Divisor.
345 | 3 Quotient.

15 5 Divisor.
4 4
60 | 20 New Divisor.
60 | 3 Quotient.

148. *If the dividend and divisor are both multiplied by the same number, the quotient remains unchanged.*

Ex. 1. Divide 91 by .7.

EXPLANATION.—We multiply both terms by 10, to make the new divisor a whole number, and divide as in integers.

SOLUTION.

Dividend. 91 ⌊ .7 Divisor.
10 10
New Dividend. 910 | 7 New Divisor.
130 Quotient.

Ex. 2. Divide 32.5 by .13.

EXPLANATION. — We multiply both terms by 100, to make the new divisor a whole number, and divide as in integers.

SOLUTION.

32.5 ⌊ .13 Divisor.
100 100
3250 | 13 New Divisor.
26 | 250 Quotient.
65
65
0

Ex. 3. Divide 6.95835 by .423.

EXPLANATION.—We multiply both terms by 1,000 by removing the decimal point three places to the right, and divide as in Case I.

SOLUTION.

6.95835 | .423 Divisor.
6958.35 | 423 New Divisor.
423 | 16.45 Quotient.
2728
2538
1903
1692
2115
2115

Ex. 4. Divide .27 by 4.32.

EXPLANATION. — We multiply both terms by 100 by removing the decimal point two places to the right, and divide as in Case I.

SOLUTION.

.27 | 4.32 Divisor.
27.00 | 432 New Divisor.
2592 | .0625 Quotient.
1080
864
2160
2160

From these examples we see that when the divisor contains one decimal figure, we multiply both terms by 10; when it contains two decimal figures, by 100; when three decimal figures, by 1,000, and so on. That is,

149. *Before dividing, both terms must be multiplied by a number composed of* 1 *with as many ciphers annexed as the divisor contains decimal figures.*

PROBLEMS.

16. If a family use .75 of a barrel of flour each month, how long will 9 barrels last them? *12 months.*

17. A farmer carried 15 bushels of wheat to mill, and received one sack of flour for every 1.25 bushels. How many sacks of flour did he get? *12.*

18. If 2.125 yards of linen can be made from .85 of a pound of flax, how much flax will be required for one yard of linen? *.4 of a pound.*

19. If a woman can weave 6.25 yards of rag carpet in one day, how long will it take her to weave 40 yards? *6.4 days.*

20. At how many loads can a teamster draw 780.7 cubic yards of gravel, if he draws .925 of a yard at each load? *844.*

21. A druggist put up .875 of gallon of sweet-oil in bottles, each containing .0625 of a gallon. How many bottles did he fill? *14.*

22. A merchant tailor sold 21 yards of silk, in vest patterns of .875 of a yard each. How many patterns did he sell? *24.*

23. If a farm hand can plow 13.5 acres of land in one week, how long will it take him to plow 47.25 acres? *3.5 weeks.*

24. What is the length of a lane which contains 21.12 square rods and is .8 of a rod wide? *26.4 rods.*

25. What is the quotient of 32.625 divided by 43.5? *.75.*

26. If a miller makes one barrel of flour from 4.5 bushels of wheat, how many barrels will he make from 49.5 bushels?

27. A lady put up 58 quarts of strawberries in cans, putting 1.8125 quarts in each. How many cans did she fill? *32.*

28. If one suit of clothes can be made from 5.75 yards of cloth, how many suits can be made from 109.25 yards? *19.*

150. *Rule for Division of Decimals.*

I. When the divisor is an integer.

1. *If necessary, annex decimal ciphers to the dividend till the figures of the dividend will contain the divisor.*

2. *Divide as in whole numbers.*

3. *Place the decimal point in the quotient so that it shall contain as many decimal figures as the dividend.*

II. When the divisor is a decimal or a mixed number.

1. *Omit the decimal point from the divisor, and remove the decimal point in the dividend as many places to the right as the original divisor contains decimal figures.*

2. *Divide and place the decimal point in the quotient as before.*

PROBLEMS.

29. Divide 7 by 43.75. *Quotient, .16.*

30. Divide 525 by 9.375. *Quotient, 56.*

31. If .625 of a yard of cloth be made from one pound of wool, how many pounds of wool will be required for 12 yards of cloth? *19.2.*

32. A farmer cut 639 cords of wood from 11.25 acres of woodland. How many cords was that to the acre? *56.8.*

33. In how many hours can you empty a cistern that contains 204 barrels of water, if you pump out 18.75 barrels each hour?

34. If a steamboat runs 156.25 miles in a day, in how long a time will it run 80 miles? *.512 of a day.*

35. How much land will be required to raise 21 bushels of corn, if the yield is at the rate of 65.625 bushels per acre?

36. If 112.59 pounds of maple sugar are made from 625.5 gallons of sap, how much sugar can be made from one gallon of sap? *.18 of a pound.*

37. The dividend is 28 and the divisor .64. What is the quotient? *43.75.*

38. What is the quotient of $85 \div .272$? *312.5.*

39. Divide 267.66 by 11.896. *Quotient, 22.5.*

40. A perfumer put up 73 gallons of cologne in bottles, putting .0625 of a gallon into each. How many bottles did he fill? *1168.*

41. How many cars will be required to transport 80410.5 tons of freight, allowing each car to carry 6.2625 tons? *12840.*

42. How many blocks of marble each weighing .9376 of a ton will together weigh 15.9392 tons? *17.*

43. If one gallon of alcohol can be made from .38 of a bushel of corn, how many gallons can be made from 15.39 bushels? *40.5.*

44. In a fence .9 of a mile long how many lengths are there, each length being .00225 of a mile long? *400.*

45. If .0196 of a cord of wood will make one bushel of charcoal, how many bushels will .833 of a cord make? *42.5.*

CASE III.

True Remainders.

151. Ex. How many barrels each holding 3.5 bushels can be filled from 237 bushels of apples, and how many apples will be left?

SOLUTION.

237 | 3.5
2370 | 35
210 | 67 barrels.
270
245
25 Remainder.
2.5 True remainder.

EXPLANATION. — Since the divisor contains one decimal figure, we multiply both terms by 10 before commencing to divide. But since 2,370, the dividend used, is 10 times as great as the given dividend, the remainder 25, which is a part of this 2,370, is ten times as great as the *true remainder*. Therefore, to find the true remainder, we divide the 25 by 10, which we do by placing a decimal point before the 5. Hence, 67 barrels and 2.5 bushels over is the result required.

152. *When the quotient is an integer, the true remainder must always contain as many decimal figures as there are in the given term having the more decimal figures.*

PROBLEMS.

46. A farmer has 494 gallons of cider. How many barrels can he fill putting 31.5 gallons into each?

15, and have 21.5 gallons left.

47. If 9.5 tons of freight make 1 car load, how many car loads are 124.2 tons? *.7 of a ton more than 13 car loads.*

48. A farmer who has 134 bushels of wheat, wishes to exchange it for sheep. If he gives 2.5 bushels for 1 sheep, how many sheep can he buy? *53, and have 1.5 bushels of wheat left.*

49. Into how many building lots each containing 1.25 acres can I divide 9.5 acres? *Into 7, and .75 of an acre remaining.*

50. A quartermaster has 834.25 pounds of coffee. If he distributes 71.25 pounds to his regiment daily, how many days' rations of coffee has he, and how much over?

11 days' rations, and 50.5 pounds over.

SECTION VI.

UNITED STATES MONEY.

153. United States Money — also called *Federal Money*—consists of dollars, cents, and mills.

10 mills are 1 cent.	1 dollar is 100 cents.
100 cents are 1 dollar.	1 cent is 10 mills.

154. Since 100 cents are 1 dollar, 1 cent is 1 hundredth of a dollar, and is written $.01. And

155. Since 10 mills are 1 cent, 1 mill is 1 tenth of a cent or 1 thousandth of a dollar, and is written $.001.

The divisions of a dollar into cents and mills correspond to the decimal divisions of a dollar into hundredths and thousandths. Hence,

156. *Cents may always be written as hundredths, and mills as thousandths, of a dollar.*

35 cents are written $.35.
8 cents are written $.08. | 6 mills are written $.006.
10 cents 5 mills are written $.105.
7 dollars 93 cents are written $7.93.
5 dollars 56 cents 8 mills are written $5.568.
20 dollars 20 cents 1 mill are written $20.201.

EXERCISES.

1. Read $.15, $.60, $318.75, $14.06, $5.94, $8.01.
2. Read $.255, $.004, $300.567, $12.108, $575.10, $900.25.
3. Write 37 cents, 80 cents, 6 cents.
4. Write 13 dollars 4 cents, 75 dollars 50 cents.
5. Write 8 mills, 15 cents 6 mills.
6. Write 83 dollars 12 cents 5 mills.
7. Write 400 dollars 8 cents 1 mill.

157. *Decimal parts of a dollar less than mills or thousandths are read as decimals of a mill.*

$.0006 is 6 tenths of a mill; $.0085 is 8 and 5 tenths mills.
$.2943 is 29 cents 4 and 3 tenths mills.
$15.65425 is 15 dollars 65 cents 4 and 25 hundredths mills.

COMPUTATIONS OF U. S. MONEY.

158. Ex. 1. What is the sum of \$108.50, \$10.875, and \$.458?

SOLUTION.

\$108.50
10.875
.458
\$119.833

EXPLANATION. — We write the numbers with dollars under dollars, cents under cents, and mills under mills; and then add the parts, and place the decimal point in the sum, as in Addition of Decimals.

Ex. 2. From \$45.25 subtract \$17.625.

SOLUTION.

\$45.250
17.625
\$27.625

EXPLANATION.—We write the numbers with dollars under dollars, cents under cents, and mills under mills; and then subtract, and place the decimal point in the remainder, as in Subtraction of Decimals.

Ex. 3. Multiply \$8.125 by 2.7.

SOLUTION.

\$8.125
2.7
56875
16250
\$21.9375

EXPLANATION. — We write the multiplier under the multiplicand; and then multiply, and place the decimal point in the product, as in Multiplication of Decimals.

Ex. 4. Divide \$436.72 by 53.

SOLUTION.

\$436.72 | 53
424 | \$8.24
127
106
212
212

EXPLANATION.—We write the divisor at the right of the dividend; and then divide, and place the decimal point in the quotient, as in Division of Decimals.

Ex. 5. How many times are \$27.50 contained in \$1168.75?

SOLUTION.

\$1168.75 | \$27.50
116875 | 2750
11000 | 42.5
6875
5500
13750
13750

159. In business, when the mills in any final result are 5 or more, they are regarded as 1 cent, and when less than 5, they are rejected.

PROBLEMS.

Find the sum of the several amounts of money in the first four problems.

(1)	(2)	(3)	(4)
$121.10	$ 7.28	$.58	$2000
38.47	241.09	.145	5.75
92.86	.42	.0275	48.01
582.79	.96	.5625	.495
810.04	44.52	.095	359.17

5. One day a toll-gate keeper received $17.56, and the next day $28.25. How much toll did he receive in the two days?

6. A furniture dealer sold a wash-stand for $6.50, a bureau for $11.63, and a rocking-chair for $8.25. For how much did he sell all of them? *$26.38.*

7. On Saturday night a laborer paid $1.625 for flour, $.85 for tea, $.75 for sugar, and $.375 for butter. How much money did he pay out? *$3.60.*

	(8)	(9)	(10)	(11)
From	$250.35	$.104	$100.000	$1.000
take	187.50	.087	5.875	.065

12. I owe $167.45. If I pay $94.50, how much will I then owe? *$72.95.*

13. A lawyer having $256.56 in the bank, drew out $98.75. How much money had he left in the bank? *$157.81.*

14. One week a laborer earned $12.50, and expended $8.38. How much of his earnings had he left? *$4.12.*

	(15)	(16)	(17)	(18)
Multiply	$194.17	$310.75	$50.44	$249.60
by	8	36	7.5	.25

19. How much will 7 bushels of wheat cost at $1.125 a bushel? *$7.875.*

20. If an ounce of indigo costs $.15, how much will 9 ounces cost? *$1.35.*

21. A builder bought 37 thousand feet of pine lumber at $28.25 a thousand. How much did it cost him? *$1045.25.*

22. How much will 8.5 gallons of kerosene cost at $.625 a gallon? *$5.3125.*

Find the quotient in problems 23, 24, 25, and 26.

(23)	(24)	(25)	(26)
$51.75 ⌊ 9	$156.06 ⌊ 8.67	$405.65 ⌊ $21.35	$7.82 ⌊ $.085

27. If 7 pounds of sugar cost $.91, what is the price of a pound? *$.13.*

28. At what price per head must I sell 105 sheep, to receive $564.375 for them? *$5.375.*

29. A grocer paid $35 for a barrel of sugar at $.125 a pound. How many pounds did he buy? *280.*

30. A chair maker received $172.50 for chairs at $7.50 a set. How many sets did he sell? *23.*

31. If 31.5 gallons of linseed-oil cost $59.0625, what is the price of a gallon? *$1.875.*

160. *Rule for Computations of United States Money.*

Write the numbers, and add, subtract, multiply, divide, and place the decimal points in the results, as in Decimals.

PROBLEMS.

32. A man bought a village lot for $325, and after paying $22.63 for taxes, he sold it so as to gain $72.37. For how much did he sell it? *$420.*

33. I bought a hat for $4.875, a coat for $28, and a pair of boots for $7.50. How much did they cost me? *$40.375.*

34. A farmer sold a jar of butter to a merchant for $10.37, receiving in payment groceries to the amount of $4.63, and the balance in money? How much money did he receive? *$5.74.*

35. How much will 125 pounds of nails cost at $.06 a pound? *$7.50.*

36. What will be the cost of .84 of a ton of plaster at $4.25 a ton? *$3.57.*

37. At $2.50 a bushel, how many plums can be bought for $1.875? *.75 of a bushel.*

38. How much will 29 rolls of wall-paper cost at $.44 a roll? *$12.76.*

39. How much muslin can be bought for $24.36, at $.56 a yard? *43.5 yards.*

40. I paid $.78 for a piece of fresh beef at $.12 a pound. How much did it weigh? 6.5 *pounds.*

41. A mechanic earned $56.25 in January, $45.63 in February, $67.50 in March, $65.875 in April, and $75 in May. How much did he earn in the five months? *$310.255.*

42. Mr. Stevens bought a watch for $32.25, and sold it to Mr. Adams for $27.75. How much did he lose by the transaction? *$4.50.*

43. Mr. Adams afterward sold it for $30.625. How much did he gain? *$2.875.*

44. One season a nurseryman sold 2840 young apple-trees at $.375 apiece. How much did he receive for them? *$1065.*

45. A stationer paid $114 for pocket-knives at $.95 apiece. How many did he buy? *120.*

46. How much must be paid for transporting .456 of a ton of freight from New York to Toledo by railroad, at $28.60 a ton? *$13.0416.*

47. A fruit dealer sold 686 baskets of peaches for $1543.50. What was the price per basket? *$2.25.*

SECTION VII.

PROBLEMS IN DECIMALS.

1. A merchant deposited $59.17 in the bank on Monday, $62.86 on Tuesday, $48.12 on Wednesday, $75.48 on Thursday, $88.57 on Friday, and $110.72 on Saturday. What was the amount of his deposits for the week? *$444.92.*

2. What will be the cost of 15,000 bushels of wheat at $2.0625 a bushel? *$30937.50.*

3. When the price of rice is $.0625 a pound, how many pounds can be bought for $3.50? *56.*

4. If wheat is worth $1.4375 per bushel in Chicago, and $2.125 in New York, how much is added to its value by transportation? *$.6875 per bushel.*

5. How much will it cost to build 18.4 rods of picket-fence at $3.125 a rod? *$57.50.*

6. A fruit dealer paid $14.25 for 95 quarts of strawberries. How much did he pay a quart for them? *$.15.*

7. I cut .912 of a ton of hay from my door yard, and the yield was at the rate of 1.92 tons to the acre. How much land is there in my door yard? *.475 of an acre.*

8. When cranberries are worth $5.00 a bushel, what part of a bushel can be bought for $.625? *.125.*

9. A shoemaker paid $385.40 for sole-leather, $216.94 for upper-leather, $104.05 for linings, $24.28 for thread and silk, and $12.75 for pegs. How much did he pay out for stock?

10. What is the quotient of .016 ÷ .512? *.03125.*

11. When the dividend is .01 and the divisor 12.8, what is the quotient? *.00078125.*

12. Mr. Butler had a silver watch worth $18.75, which he exchanged for a gold watch worth $80, paying the balance in money. How much did he pay to boot? *$61.25.*

13. If you pay $24 for 7.5 reams of letter-paper, how much does it cost you a ream? *$3.20.*

14. How many packages each containing .875 of a pound can be filled from a chest which contains 55 pounds of tea, and how much tea will be left? *62 packages; .75 of a pound left.*

15. A man on a journey paid $32.17 for railroad fare, $12.44 for steamboat fare, $37.25 for hotel bills, and $7.32 for other expenses. What were his total expenses? *$89.18.*

16. How much hemlock bark at $8.25 a cord will pay a bill of $57.75 for boots and shoes? *7 cords.*

17. At $4.375 a head, how much will 1000 sheep cost?

18. A merchant's sales in September were $2174.15, in October $1416.24, in November $1765.93, and in December $2443.76. How much did his sales average per month?

19. A man who owed $250, paid at one time $65.48, at another time $47.81, and at another $92.37. How much did he then owe? *$44.34.*

20. What is the quotient of .315 ÷ .3125? *1.008.*

21. If my expenses for five consecutive weeks are $12.25, $13.61, $14.09, $11.52, and $13.78, how much are my average weekly expenses? *$13.05.*

22. A fruit dealer paid $15.45 for oranges, $20.34 for lemons, $27.59 for pine-apples, and $16.72 for cocoa-nuts. How much did the fruit cost him? *$80.10.*

23. If I buy goods to the amount of $8.45, and in paying for them give a 10-dollar bill, how much change ought I to receive? *$1.55.*

24. A merchant tailor sold a piece of damaged cloth at a loss of $19.88, and the cloth cost him $87.50. For how much did he sell it? *$67.62.*

25. A dealer in agricultural implements paid $199.50 for plows, at $7.125 each. How many plows did he buy? *28.*

26. What will be the cost of 32.5 yards of tapestry carpet at $2.75 a yard? *$89.375.*

27. If 1 coat can be made from 3.125 yards of cloth, how many can be made from 52.5 yards?

16, with a remnant of 2.5 yards.

28. A spice dealer put up 280 pounds of ground cinnamon in boxes, each holding .25 of a pound. How many boxes did he fill? *1120.*

29. From a piece of cloth containing 44.5 yards, a tailor made as many suits of clothes, each containing 8.375 yards, as he could. How many yards were left in the piece? *2.625.*

30. How many pounds of metal will it take to cast one church bell weighing 2,765 pounds, and 7 factory bells, each weighing 325 pounds? *5,040.*

31. From a cask which contained 37.175 gallons of alcohol, a druggist drew, at different times, .125 of a gallon, 1.5 gallons, .25 of a gallon, 2.75 gallons, .625 of a gallon, .0625 of a gallon, and .75 of a gallon. How many gallons were then left in the cask? *31.1125.*

32. How much delaine at $.5625 a yard can be bought for $9? *16 yards.*

33. A butcher paid $58.60 for an ox, and after killing it, he retailed the meat for $59.76, sold the tallow for $4.18, and the hide for $7.88. How much were his profits? *$13.22.*

34. A provision dealer bought pork at $.125 a pound, and sold it at $.13. How much did he gain a pound?

35. If a family use .785 of a barrel of flour in one month, .825 of a barrel the next month, .73 of a barrel the third, .8 of a barrel the fourth, and .76 of a barrel the fifth, what is the average amount used per month? *.78 of a barrel.*

36. A farmer harvested 713.5 bushels of oats from a field of 13 acres, and 576.25 bushels from a field of 9 acres. What was the average yield per acre of the two fields? *58.625 bushels.*

37. 11 miles of a certain railroad cost $13875.30 per mile, 14 miles cost $15251.64 per mile, and 28 miles cost $14588.45 per mile. What was the length of the road, and what the average cost per mile? *Cost per mile, $14615.62.*

38. George Wells bought 5 yards of casimere at $1.875, 1 yard of alpaca for $.875, 13 yards of calico at $.25, and 14 yards of muslin at $.35. What was the cost of the whole?

(See Manual, page 219.)

39.

Newark, May 3, 1867.

Mr. Robert Newton,

Bought of F. B. Adams & Co.

5 papers Garden Seeds,	*@ $.08,*	$ .40
3 quarts Marrowfat Peas,	*" .15,*	.45
1 sack Flour,		3.20
1 Garden Rake,		.75
1 Plow, $9.25; 1 Spade, $1.25,		10.50
2 Hoes, $.69 and $.75,		1.44
		$16.74

Rec'd Payment,

F. B. Adams & Co.

CHAPTER III.

COMPOUND NUMBERS.

SECTION I.

NOTATION AND REDUCTION.

161. We find the amount or quantity of articles bought and sold, by measuring or weighing them.

Some articles are sold by the quart or gallon ; some by the peck or bushel ; some by the foot or yard ; some by the acre ; some by the cord, and some by the pound or ton.

162. The names applied to particular amounts or quantities of articles are *Denominations ;* as the gallon, bushel, foot, mile, pound, and dozen.

163. Numbers applied to denominations are *Denominate Numbers ;* as 4 yards, 9 bushels.

164. A number expressed in two or more denominations is a *Compound Number ;* as 4 hours 30 minutes, 3 yards 2 feet 6 inches.

A denominate number may be an integer, as 3 bushels ; a decimal, as .5 of a mile ; a mixed number, as 6.75 tons, or a compound number, as 5 pounds 6 ounces.

In writing compound numbers, the denominations are generally abbreviated, as shown in the tables.

(See Manual, page 219.)

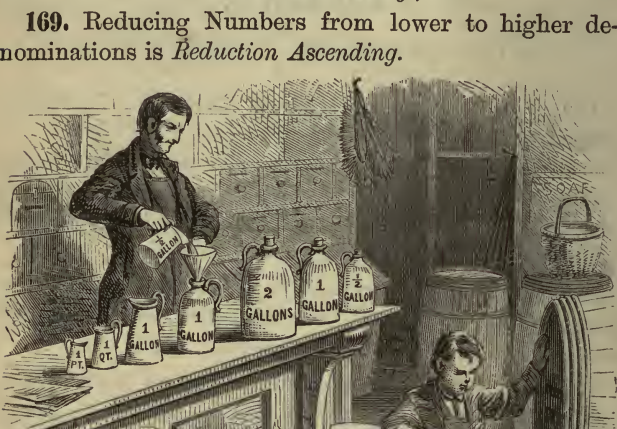
165. Those denominations in a compound number which express the greater amount are *Higher Denominations :* and

166. Those which express the less amount are *Lower Denominations.* Thus, a quart is a higher denomination than a pint, and a lower denomination than a gallon.

167. Changing numbers from one denomination to another without changing their value is *Reduction.*

168. Reducing numbers from higher to lower denominations is *Reduction Descending;* and

169. Reducing Numbers from lower to higher denominations is *Reduction Ascending.*



170. *Table I.—Liquid Measure.*

The denominations gallons, quarts, pints, and gills constitute *Liquid Measure.* They are used in measuring oil, molasses, wines, milk, and other liquids.

4 gi. (gills)	are 1 pt. (pint).	1 gal.	is 4 qt.
2 pt.	" 1 qt. (quart).	1 qt.	" 2 pt.
4 qt.	" 1 gal. (gallon).	1 pt.	" 4 gi.

EXERCISES.

1. Read 5 gal. 3 qt. 1 pt. 1 gi.; 14 gal. 3 qt. 1 pt.
2. Read 11 gal. 1 pi. 2 gi.; 7 gal. 1 pt.; 3 qt. 1 gi.
3. Write five gallons one quart one pint two gills.
4. Write seventeen gallons two quarts three gills.

REDUCTION DESCENDING.

171. Ex. 1. How many pints are equal to 3 gallons?

SOLUTION.

3 *gal.*
4
12 *qt.*
2
24 *pt.*

EXPLANATION.—Since 3 gal. are 3 times 1 gal., and 1 gal. is 4 qt., 3 gal. are 3 times 4 qt., or 12 qt.; and since 12 qt. are 12 times 1 qt., and 1 qt. is 2 pt., 12 qt. are 12 times 2 pt., or 24 pt.

Hence, *3 gal.* = *24 pt.*

Ex. 2. How many pints are equal to 2 gal. 3 qt. 1 pt.?

EXPLANATION.—Since 2 gal. are 2 times 1 gal., and 1 gal. is 4 qt., 2 gal. are 2 times 4 qt., or 8 qt., and 8 qt. + 3 qt. are 11 qt. Since 11 qt. are 11 times 1 qt., and 1 qt. is 2 pt., 11 qt. are 11 times 2 pt., or 22 pt., and 22 pt. + 1 pt. are 23 pt.

SOLUTION.

2 *gal.* 3 *qt.* 1 *pt.*
4
8 + 3 = 11 *qt.*
2
22 + 1 = 23 *pt.*

Hence, *2 gal. 3 qt. 1 pt.* = *23 pt.*

PROBLEMS.

1. Reduce 11 gallons to quarts. (See Ex. 1.) *44 qt.*
2. How many gills are there in 4 quarts? *32.*
3. A wholesale druggist put ten gallons of sweet-oil into bottles which held 1 gill each. How many bottles did he fill? *320.*
4. How many pint bottles will be required to hold 7 gallons of currant wine? *56.*
5. In 2 gal. 3. qt. 1 pt. 3 gi., how many gills? (See Ex. 2.) *95.*

6. A druggist has 11 gal. 2 qt. of alcohol. How long will it last him, if he sells 1 qt. each day? *46 days.*

7. How many pint bottles will 4 gal. 2 qt. 1 pt. of catchup fill?

8. A grocer bought a barrel containing 31.5 gallons of vinegar, which he sold by the quart. How many quarts did he sell? *126.*

9. Reduce 5 gal. 2 qt. 1 pt. 2 gi. to gills.

REDUCTION ASCENDING.

172. Ex. 1. How many gallons are 48 pints?

EXPLANATION. — Since every 2 pt. are 1 qt., and 2 pt. are contained in 48 pt. 24 times, 48 pt. are 24 qt. And since every 4 qt. are 1 gal., and 4 qt. are contained in 24 qt. 6 times, 24 qt. are 6 gal.

SOLUTION.

48 *pt.* ⌊ 2 *pt.*

24 *times.*

48 *pt.* = 24 *qt.*

24 *qt.* ⌊ 4 *qt.*

6 *times.*

24 *qt.* = 6 *gal.*

Hence, *48 pt.* = *6 gal.*

Ex. 2. How many gallons in 79 pints?

EXPLANATION. — Since every 2 pt. are 1 qt., and 2 pt. are contained in 79 pt. 39 times with a remainder of 1 pt., 79 pt. are 39 qt. 1 pt. And since every 4 qt. are 1 gal., and 4 qt. are contained in 39 qt. 9 times with a remainder of 3 qt., 39 qt. are 9 gal. 3 qt.

In the first, or Full Solution, we have written all the numbers mentioned in the explanation, both

FULL SOLUTION.

79 *pt.* ⌊ 2 *pt.*

39 *times and* 1 *pt. rem.*

79 *pt.* = 39 *qt.* 1 *pt.*

39 *qt.* ⌊ 4 *qt.*

9 *times and* 3 *qt. rem.*

39 *qt.* = 9 *gal.* 3 *qt.*

Hence, *79 pt.* = *9 gal. 3 qt. 1 pt.*

COMMON SOLUTION.

79 *pt.* ⌊ 2 *pt.*

39 *qt.* 1 *pt.* ⌊ 4 *qt.*

9 *gal.* 3 *qt.*

Hence, *79 pt.* = *9 gal. 3 qt. 1 pt.*

abstract and concrete ; but in the second, or Common Solution, we have omitted the abstract quotients, and written only the denominate numbers.

PROBLEMS.

10. How many gallons are 256 quarts ? (See Ex. 1.)

11. How many gallons of cider will it take to fill 104 pint bottles ? *13.*

12. Reduce 160 gills to gallons. *5 gal.*

13. Reduce 140 gills to higher denominations. (See Ex. 2.) *4 gal. 1 qt. 1 pt.*

14. A woman buys of a milkman 1 pt. of milk a day. How much does she buy in a year or 365 days ? *45 gal. 2 qt. 1 pt.*

15. In one week a grocer sold 190 quart cans of oysters. How many gallons of oysters did he sell ? *47.5.*

16. Reduce 655 gills to higher denominations.

17. One morning a farm hand, in watering cattle, pumped 879 strokes, and at each stroke the pump discharged 1 pint of water. How much water did he pump ? *109 gal. 3 qt. 1 pt.*

We have now learned that gallons are reduced to quarts, quarts to pints, and pints to gills, by multiplying ; and that gills are reduced to pints, pints to quarts, and quarts to gallons, by dividing.

Since the reduction of gallons to gills is from a higher to a lower denomination, and the reduction of gills to gallons is from a lower to a higher denomination, the explanations already given are sufficient to establish the following

173. *General Principles of Reduction.*

I. *A denominate number is reduced to lower denominations by multiplication.*

II. *A denominate number is reduced to higher denominations by division.*

174. ***Table II.—Dry Measure.***

The denominations bushels, pecks, quarts, and pints constitute *Dry Measure.* They are used in measuring grain, seeds, fruits, berries, several kinds of vegetables, lime, charcoal, and some other articles.

In measuring grain, seeds, peas, beans, and small fruits, the measure must be *even* full. But in measuring large fruits, coarse vegetables, and other bulky articles, the measure must be *heaping* full. 4 heaped measures must equal 5 even measures.

2 pt. are 1 qt.	1 bu. is 4 pk.
8 qt. " 1 pk. (peck.)	1 pk. " 8 qt.
4 pk. " 1 bu. (bushel.)	1 qt. " 2 pt.

The quart and pint of Dry Measure are larger than the quart and pint of Liquid Measure. 6 quarts Dry Measure are equal to nearly 7 quarts Liquid Measure.

EXERCISES.

1. Read 17 bu. 1 pk. 2 qt. 1 pt.; 2 pk. 1 pt.
2. Read 19 bu. 5 qt.; 3 pk. 4 qt. 1 pt.
3. Write one bushel two pecks four quarts one pint.
4. Write five bushels six quarts one pint.
5. Write twenty-eight bushels three pecks.

175. Ex. 1. Reduce 13 bu. 3 qt. to quarts.

SOLUTION.

13 *bu.* 0 *pk.* 3 *qt.*
4
52 *pk.*
8
416 + 3 = 419 *qt.*

Hence, *13 bu. 3 qt. = 419 qt.*

EXPLANATION.—Since there are no pecks, while there are denominations both higher and lower than pecks, we write a 0 in the place of pecks in the Solution.

Ex. 2. Reduce 195 qt. to bu.

SOLUTION.

195 *qt.* ⌊ 8 *qt.*
24 *pk.* 3 *qt.* ⌊ 4 *pk.*
6 *bu.*

Hence, *195 qt. = 6 bu. 3 qt.*

EXPLANATION. — In reducing 24 pecks to bushels, we have 0 pecks remaining. There are no pecks, therefore, in the final result.

176. *Rules for Reduction.*

I. For Reduction Descending.

1. *Multiply the highest denomination given by that number of the next lower denomination which equals* 1 *of this higher, and to the product add the given lower denomination.*

2. *In the same manner, reduce this result to the next lower denomination; and so continue until the given number is reduced to the required denomination.*

II. For Reduction Ascending.

1. *Divide the given denomination by that number of this denomination which equals* 1 *of the next higher, writing the quotient as so many of the higher denomination, and the remainder as so many of the denomination divided.*

2. *In the same manner, reduce this quotient to the next higher denomination; and so continue until the given number is reduced to the required denomination.*

3. *Write the last quotient and the several remainders in their order, from the highest denomination to the lowest, for the required result.*

PROBLEMS.

18. How many quarts are there in 18 bu. 2 pk. 3 qt.? *595.*

19. How long will 2 bu. 3 pk. 4 qt. of corn last my hens, if I feed them 1 pt. each day? *184 days.*

20. How many pint papers of seed-corn are equal to 7 bu. 3 pk. 5 qt. 1 pt.?

21. Reduce 553 pints to higher denominations.

22. A farmer's boy fed to his colt 1 pint of oats each day for 150 days. How many oats did he feed? *2 bu. 1 pk. 3 qt.*

23. A blackberry girl sold 10 quarts of blackberries each day for 18 days. How many berries did she sell?

5 bu. 2 pk. 4 qt.

24. Reduce 275 bu. 7 qt. to quarts. *8,807 quarts.*

25. A gardener put 4 bu. 5 qt. of strawberries into quart boxes. How many boxes did he fill? *133.*

26. How many times can a pint measure be filled from 3 bu. 2 pk. 1 pt. of chestnuts? *225.*

27. Reduce 261 quarts to higher denominations. *8 bu. 5 qt.*

28. A dealer in garden seeds put up 353 pint papers of marrowfat peas. How many peas did he put up?

5 bu. 2 pk. 1 pt.

29. If 1 peck of clover seed will seed one acre of land, how much land can a farmer seed with 5.5 bushels? *22 acres.*

177. *Table III.—Linear Measure.*

The denominations miles, rods, yards, feet, and inches constitute *Linear* or *Line Measure.* They are used in measuring distances, and also the length, width, thickness, height, and depth of things.

This line ——— is one inch long.

12	in. (inches)	are	1 ft.	(foot.)	1 mi.	is 320	rd.
3	ft.	"	1 yd.	(yard.)	1 rd.	" 5.5	yd.
5.5	yd.	"	1 rd.	(rod.)	1 yd.	" 3	ft.
320	rd.	"	1 mi.	(mile.)	1 ft.	" 12	in.

In this picture the gateway is represented as 1 rod wide; the board fence as 40 rods, or 1 eighth of a mile long; the large tree as 80 rods, or 1 fourth of a mile from the corners; the corners as 160 rods, or 1 half mile from the gate; and the house as 1 mile from the gate.

EXERCISES.

1. Read 4 mi. 114 rd. 4 yd. 1 ft. 10 in.; 17 mi. 46 rd. 2.5 ft.

2. Write eight miles twenty-six rods two yards two feet six inches.

3. Write three hundred nineteen miles sixty-seven rods three and seventy-five hundredths yards.

PROBLEMS.

30. How many feet are 63 rd. 4 yd. 2 ft.?

31. How many blocks each 1 inch long can be cut from a board 15 feet long? *180.*

32. Reduce 5 mi. 187 rd. 2 yd. 1 ft. 9 in. to inches. *353,919 inches.*

SOLUTION OF PROBLEM 30.

63 *rd.* 4 *yd.* 2 *ft.*

5.5

315

315

346.5 + 4 = 350.5 *yd.*

3

1051.5 + 2 = 1053.5 *ft.*

Hence, *63 rd. 4 yd. 2 ft. = 1053.5 ft.*

33. How many feet are 200 mi. 4 yd.? *1,056,012.*

34. How many rods of fence will it take to inclose a farm which is 1 mile long and .5 of a mile wide? *960.*

35. Reduce 398 yards to rods.

SOLUTION OF PROBLEM 35.

398 *yd.* ⌊ 5.5 *yd.*

3980 ⌊ 55

385 ⌊ 72 *rd.*

130

110

2.0 *yd.*

Hence, *398 yd. = 72 rd. 2 yd.*

36. In a bundle of lath there are 100 pieces, each 4 feet long. If laid lengthwise in a row upon the ground, how far would they reach? *24 rd. 1 yd. 1 ft.*

37. Reduce 1,530 inches to higher denominations. *7 rd. 4 yd.*

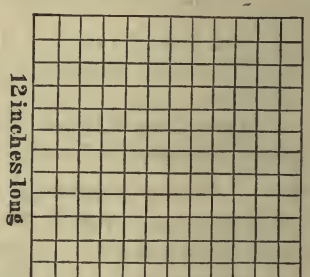
38. How many tiles each 1 foot long will be required for 1 mi. 68 rd. 2 yd. of tile-drain? *6,408.*

39. How many miles are there in the fences that inclose the farm shown in the map on page 25? *2 mi. 242 rd.*

178. *Table IV.—Square Measure.*

The denominations square miles, acres, square rods, square yards, square feet, and square inches constitute *Square Measure.* They are used in measuring land, flooring, plastering, and other surfaces.

A square foot is 1 foot or 12 inches long, and 1 foot or 12 inches wide. Hence, it contains 12 times 12, or 144 square inches.



144	sq. in. (square in.)	are 1	sq. ft.	1 sq. mi. is 640	A.
9	sq. ft.	" 1	sq. yd.	1 A. " 160	sq. rd.
30.25	sq. yd.	" 1	sq. rd.	1 sq. rd. " 30.25	sq. yd.
160	sq. rd.	" 1	A. (acre.)	1 sq. yd. " 9	sq. ft.
640	A.	" 1	sq. mi.	1 sq. ft. " 144	sq. in.

EXERCISES.

1. Read 14 sq. mi. 84 A. 28 sq. rd.

2. Read 25 sq. rd. 16 sq. yd. 84 sq. in.

3. Write two hundred nine square miles eighty six acres one hundred seven square rods.

4. Write five square yards eight square feet thirty-six square inches.

PROBLEMS.

40. Reduce 34 square miles to square rods. *3,481,600 sq. rd.*

41. A farmer planted one hill of corn upon every square yard of a 10-acre lot. How many hills did he plant? *48,400.*

42. Reduce 84 sq. rd. 4 sq. ft. to square feet. *22,873 sq. ft.*

43. In 25.3 square miles how many acres? *16,192 A.*

44. How many square miles are there in 312,000 square rods?

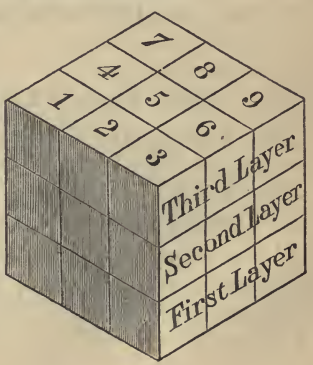
45. A fruit grower has an orchard containing 6,386 peach trees, and each tree occupies 1 square rod of land. How much land is there in the orchard? *39 A. 146 sq. rd.*

46. In covering a roof a tinsmith used 1,152 sheets of tin, each covering 14 by 20 inches. How many square feet were in the roof? *2,240.*

47. Reduce 334,976 square inches to higher denominations. *8 sq. rd. 16 sq. yd. 4 sq. ft. 32 sq. in.*

179. *Table V.—Cubic Measure.*

The denominations cubic yards, cubic feet, and cubic inches constitute *Cubic Measure.* They are used in measuring earth, timber, stone, and many other articles, and in estimating the capacity of bins, boxes, etc.



A cubic foot is 1 foot or 12 inches long, 1 foot or 12 inches wide, and 1 foot or 12 inches

thick, and hence it contains 12 times 12 times 12, or 1728, cubic inches. A cubic yard is 3 feet long, 3 feet wide, and 3 feet thick, and contains 27 cubic feet.

1728 cu. in. (cubic inch) are 1 cu. ft.	1 cu. yd. is 27 cu. ft.
27 cu. ft. " 1 cu. yd.	1 cu. ft. " 1728 cu. in.

EXERCISES.

1. Read 30 cu. yd. 10 cu. ft. 1008 cu. in.
2. Read 215 cu. yd. 49 cu. in.
3. Write fourteen cubic yards twenty-four cubic feet.
4. Write one hundred nine cubic yards ninety-two cubic inches.

PROBLEMS.

48. In 3 cu. yd. 17 cu. ft. 112 cu. in., how many cubic inches?
49. Reduce 846,296 cubic inches to higher denominations. *18 cu. yd. 3 cu. ft. 1304 cu. in.*
50. Reduce 5 cu. yd. 948 cu. in. to cubic inches. *234,228.*
51. How many cubical blocks, each containing 1 cubic inch, will be required to make a pile that shall contain 16.5 cubic yards? *769,824.*
52. A brick is 8 inches long, 4 inches wide, and 2 inches thick. If 100,000 bricks are piled together, how many cubic yards will there be in the pile? *137 cu. yd. 4 cu. ft. 1216 cu. in.*
53. Reduce 4,713,256 cubic inches to higher denominations. *101 cu. yd. 1000 cu. in.*

180. *Table VI.—Wood Measure.*

The denominations cords, cord feet, and cubic feet constitute *Wood Measure.* They are chiefly used in measuring wood. Rough stone is also commonly sold by the cord.

A pile of wood 8 feet long, 4 feet wide, and 4 feet high contains 1 cord; and 1 foot in length of such a pile contains 16 cubic feet, and, therefore, is 1 cord foot.

16 cu. ft. are 1 cd. ft. (cord foot.)	1 cd. is 8 cd. ft.
8 cd. ft. " 1 cd. (cord.)	1 cd. " 128 cu. ft.
128 cu. ft. " 1 cd.	1 cd. ft. " 16 cu. ft.

EXERCISES.

1. Read 7 cd. 5 cd. ft. 8 cu. ft.; 19 cd. 4 cd. ft.
2. Write twenty cords seven cord feet six cubic feet.
3. Write two hundred fifty-one cords two cord feet.

PROBLEMS.

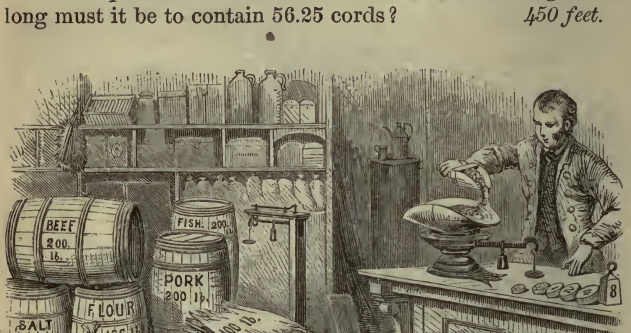
54. How many cubic feet are 13 cd. 5 cd. ft. 12 cu. ft.? *1,756.*
55. In 2,240 cubic feet of cobble stone, how many cords?

56. How many cords of wood in a pile 76 feet long, 4 feet wide, and 4 feet high? *9.5.*

57. How much wood is there in a pile 120 feet long, 4 feet wide, and 6 feet high? *22.5 cords.*

58. How much wood will a teamster draw at 10 loads, if each load is 12 feet long, 4 feet wide, and 3 feet high?

59. If a pile of wood is 4 feet wide and 4 feet high, how long must it be to contain 56.25 cords? *450 feet.*



181. *Table VII.—Weight.*

Most kinds of produce, provisions, and groceries, also iron and other metals, coal and many other articles, are bought and sold by *Weight.* The denominations in common use are tons, hundred-weight, pounds, and ounces.

16 oz. (ounces)	are 1 lb. (pound.)	1 T.	is 20 cwt.
100 lb.	" 1 cwt. (hundred-weight.)	1 cwt.	" 100 lbs.
20 cwt.	" 1 T. (ton.)	1 lb.	" 16 oz.

200 lb. of pork, beef, or fish are 1 bbl. (barrel.)
196 lb. of flour are 1 bbl.

EXERCISES.

1. Read 5 T. 17 cwt. 24 lb. 4 oz.; 39 T. 94 lb.

2. Write seven tons thirteen hundred-weight fifty-nine pounds fourteen ounces.

3. Write one ton nine hundred forty-eight pounds five ounces.

PROBLEMS.

60. How many ounces are there in 9 tons? *288,000.*

61. Reduce 4 T. 16 cwt. 83 lb. to pounds. *9,683.*

62. One month a manufacturer put up 5 T. 4 cwt. 29 lb. of saleratus in pound packages. How many packages did he put up? *10,429.*

63. How many pound bars of lead will weigh 2 T. 54 lb.?

64. How much will 6 barrels of mackerel cost at $.085 a pound? *$102.*

65. A grocer retailed 13 barrels of flour at $.055 a pound. How much did he receive for it? *$140.14.*

66. How many tons are 34,000 pounds? *17 T.*

67. If 1 oz. of lead is used in making 1 rifle ball, how much lead will be required to make 60,000 balls? *1 T. 17 cwt. 50 lb.*

68. How many barrels of flour are 6,860 pounds? *35.*

69. Reduce 64,015 oz. to higher denominations. *2 T. 15 oz.*

182. *Table VIII.—Counting.*

In counting articles for market purposes, the denominations dozen and gross are used.

12 ones are 1 doz. (dozen.)	1 gro. is 12 doz.
12 doz. " 1 gro. (gross.)	1 doz. " 12 ones.

EXERCISES.

1. Read 24 gro. 5 doz.; 29 gro. 7 doz.

2. Write seventeen gross six dozen.

3. Write fifty gross nine dozen.

PROBLEMS.

70. If a cook uses 6 eggs each day, in how many days will she use 9 doz. eggs? *18.*

71. How many steel pens are there in 7 boxes each containing 1 gro.? *1,008.*

72. How many clothes-pins in 47 doz.? *564.*

73. How many dozens are 132 brooms? *11.*

74. In one year a tailor used 23 gro. 9 doz. buttons. What number of buttons did he use? *3,420.*

75. If 18 crayons are used every week in a certain school, how many gross will be used in 40 weeks? *5.*

76. In 1,844 screws, how many gross? *12 gro. 9 doz. 8.*

183. *Table IX.—Paper.*

Paper is bought and sold by the ream, quire, and sheet.

24 sheets are 1 quire.	1 rm. is 20 quires.
20 quires " 1 rm. (ream.)	1 quire " 24 sheets.

EXERCISES.

1. Read 4 rm. 8 quires 12 sheets; 11 rm. 10 quires.

2. Write thirteen reams fifteen quires four sheets.

PROBLEMS.

77. How many sheets of paper are there in a ream of foolscap? *480.*

78. If 1 sheet of printing paper will make 4 handbills, how many bills will 5 rm. 8 quires make? *10,368.*

79. How many letters, each requiring 1 sheet, can be written on 11 rm. 12 quires of commercial note paper? *5,568.*

80. In 384 sheets of letter paper, how many quires? *16.*

81. If a lawyer uses 18 sheets of legal cap paper every day, how many reams will he use in 320 days? *12.*

82. If a merchant uses 3 quires of wrapping paper every day of the 313 week-days of the year, how much paper does he use in the year? *46 rm. 19 quires.*

184. ***Table X.—Time.***

The denominations centuries, years, months, weeks, days, hours, minutes, and seconds are used in expressing different portions of time. The day and the year are the natural divisions of time, the other denominations, except centuries, being parts of these.

60 sec. (seconds) are 1 min. (minute.)

60 min.	are	1 h. (hour.)	1 century	is	100 yr.
24 h.	"	1 da. (day.)	1 leap-year	"	52 wk. 2 da. or 366 da.
7 da.	"	1 wk. (week.)			
52 wk. 1 da. or 365 da.	"	1 common yr. (year.)	1 common yr.	"	52 wk. 1 da. or 365 da.
52 wk. 2 da. or 366 da.	"	1 leap-yr.	1 da.	"	24 h.
			1 h.	"	60 min.
100 yr.	"	1 century.	1 min.	"	60 sec.

Every fourth year from the beginning of a century is a leap-year.

EXERCISES.

1. Read 3 yr. 6 mo.; 12 h. 30 min. 15 sec.
2. Read 9 wk. 3 da. 10 h.; 4 da. 15 h. 45 min.
3. Write five hours fifteen minutes thirty seconds.
4. Write fourteen weeks six days four hours.
5. Write twenty-eight years nine months.

Seasons.		*Months.*		*Days.*
Winter.	1st mo.	January,	Jan.	31
	2d "	February,	Feb.	28
Spring.	3d "	March,	Mar.	31
	4th "	April,	Apr.	30
	5th "	May,	May.	31
Summer.	6th "	June,	June.	30
	7th "	July,	July.	31
	8th "	August,	Aug.	31
Autumn.	9th "	September,	Sept.	30
	10th "	October,	Oct.	31
	11th "	November,	Nov.	30
Winter.	12th "	December,	Dec.	31

February has 28 days in a common year, and 29 in a leap-year. (See Manual, page 219.)

PROBLEMS.

83. How many minutes in the month of January? *44,640.*

84. How many seconds are there in a common year? *31,536,000.*

85. Reduce 3 wk. 20 min. to minutes. *30,260 min.*

86. Reduce 50,400 minutes to weeks. *5 weeks.*

87. How many seconds in the three summer months? *7,948,800.*

88. How long will it take a clock to tick 1,000,000 times, if it ticks once every second? *1 wk. 4 da. 13 h. 46 min. 40 sec.*

89. How many days were there from the beginning of the year 1857 to the end of the year 1866, two of the years, 1860 and 1864, being leap-years? *3,652.*

90. In a leap-year, how many hours? *8,784.*

91. Reduce 875,665 sec. to higher denominations. *10 da. 3 h. 14 min. 25 sec.*

92. After the 9th day of October, how many hours remain in the month? *528.*

93. If you can count 75 every minute, how much time would you spend in counting 27,000,000? *35 wk. 5 da.*

185. *The Metric System of Weights and Measures.*

In the year 1866, the Congress of the United States passed a bill authorizing the use of a new system of weights and measures. In this system the principal denomination is the *Metre*, from which all the other denominations in all the tables are derived. Hence, this system is called the *Metric System.*

The principal denomination for the Measure of Surface is the *Are;* for the Measure of Capacity, the *Litre;* and for Weight, the *Gram.* (See Manual, page 219.)

The lower denominations in each table are tenths, hundredths, or thousandths of these; and their names are formed by prefixing *deci*, *centi*, or *milli* to the name of the principal denomination.

The higher denominations are 10, 100, 1,000, or 10,000 times the principal denomination of any table; and their names are formed by prefixing *deka*, *hecto*, *kilo*, or *myria* to the name of that principal denomination.

TABLE OF DENOMINATIONS AND THEIR RELATIVE VALUES.

PREFIXES FOR LOWER DENOMINATIONS.	NAMES OF PRINCIPAL DENOMINATIONS.	PREFIXES FOR HIGHER DENOMINATIONS.
Milli (mill-ee) .001 of	Metre (mee-ter)	*Deka* (dek-a) 10
Centi (sent-ee) .01 of	Are (āre)	*Hecto* (hec-to) 100
Deci (des-ee) .1 of	Litre (li-ter)	*Kilo* (kill-o) 1,000
	Gram	*Myria* (mĭr-e-a) 10,000

The weights and measures of this system have not yet come into use in this country. They are in general use in France, Belgium, Spain, and Portugal; and their use has been legalized by Great Britain, Italy, Norway, Sweden, Greece, Mexico, and most of the South American governments.

MEASURES OF LENGTH.

10 millimetres	are 1 centimetre	1 millimetre	is $\frac{1}{1000}$ metre
10 centimetres	" 1 decimetre	1 centimetre	" $\frac{1}{100}$ metre
10 decimetres	" 1 metre	1 decimetre	" $\frac{1}{10}$ metre
10 metres	" 1 dekametre	1 METRE	" *39.37 inches.*
10 dekametres	" 1 hectometre	1 dekametre	" 10 metres
10 hectometres	" 1 kilometre	1 hectometre	" 100 metres
10 kilometres	" 1 myriametre	1 kilometre	" 1,000 metres
		1 myriametre	" 10,000 metres

MEASURES OF SURFACE.

		1 centare	is $\frac{1}{100}$ are
100 centares	are 1 are	1 ARE	" 100 sq. metres, *119.6 sq. yd.*
100 ares	" 1 hectare	1 hectare	" 100 ares

MEASURES OF CAPACITY.

10 millilitres	are 1 centilitre	1 millilitre	is $\frac{1}{1000}$ litre
10 centilitres	" 1 decilitre	1 centilitre	" $\frac{1}{100}$ litre
10 decilitres	" 1 litre	1 decilitre	" $\frac{1}{10}$ litre
10 litres	" 1 dekalitre	1 LITRE	" 1 cu. decimetre, *.908 qt. dry meas.*, *1.0567 qt. liq. meas.*
10 dekalitres	" 1 hectolitre	1 dekalitre	" 10 litres
10 hectolitres	" 1 kilolitre, or stere	1 hectolitre	" 100 litres
		1 kilolitre or stere	" 1000 litres

WEIGHT.

10 milligrams	are 1 centigram	1 milligram	is $\frac{1}{1000}$ gram
10 centigrams	" 1 decigram	1 centigram	" $\frac{1}{100}$ gram
10 decigrams	" 1 gram	1 decigram	" $\frac{1}{10}$ gram
10 grams	" 1 dekagram	1 GRAM	" *15.432 grains*
10 dekagrams	" 1 hectogram	1 dekagram	" 10 grams
10 hectograms	" 1 kilogram	1 hectogram	" 100 grams
10 kilograms or kilos	" 1 myriagram	1 kilogram or 1 KILO	" 1000 grams, or *2.2046 pounds*
10 myriagrams	" 1 quintal	1 myriagram	" 10 kilos
10 quintals	" 1 millier or tonneau	1 quintal	" 100 kilos
		1 millier	" 1,000 kilos

SECTION II.

ADDITION.

186. Ex. What is the sum of 5 rd. 4 yd. 2 ft. 3 in., 7 rd. 1 yd. 1 ft. 9 in., 2 yd. 1 ft., 2 ft. 11 in., and 12 rd. 5 yd. 5 in.?

EXPLANATION.—We write the numbers so that like denominations stand in the same columns. Commencing with the lowest denomination, we add; 5 in. + 11 in. + 0 in. + 9 in. + 3 in. = 28 in. But 28 in. = 2 ft. 4 in.; we therefore write the 4 in. as the inches of the sum, and the 2 ft. we add with the feet of the given numbers. 2 ft. + 0 ft. + 2 ft. + 1 ft. + 1 ft. + 2 ft. = 8 ft. But 8 ft. = 2 yd. 2 ft; we therefore write the 2 ft. in the sum, and add the 2 yd. with the column of yards. 2 yd. + 5 yd. + 2 yd. + 1 yd. + 4 yd. = 14 yd. But since 14 yd. = 2 rd. 3 yd., we write the 3 yd. in the sum, and add the 2 rd. with the column of rods. 2 rd. + 12 rd. + 7 rd. + 5 rd. = 26 rd., which we write as the rods of the sum. The result, 26 rd. 3 yd. 2 ft. 4 in., is the sum required.

SOLUTION.

5 *rd.*	4 *yd.*	2 *ft.*	3 *in.*
7	1	1	9
	2	1	0
		2	11
12	5	0	5
26 *rd.*	3 *yd.*	2 *ft.*	4 *in.*

PROBLEMS.

Find the sum of the compound numbers in problems 1, 2, 3.

(1)		(2)			(3)		
9 *A.*	96 *sq. rd.*	3 *rm.*	5 *quires*	16 *sheets*	34 *gal.*	2 *qt.*	0 *pt.*
11	44	4	0	8	35	1	1
8	108	3	20	6	33	3	1
10	56	2	18	14	36	1	0

4. A painter used 5 gal. 3 qt. 1 pt. of linseed-oil one week, and 3 gal. 2 qt. 1 pt. the next week. How much did he use in the two weeks? *9 gal. 2 qt.*

5. A farmer used 2 bu. 1 pk. 6 qt. of clover seed in seeding one field, and 3 bu. 3 pk. 4 qt. in seeding another. How much did he use upon the two fields? *6 bu. 1 pk. 2 qt.*

6. How much wood is there in three piles, the first of which contains 5 cd. 3 cd. ft. 12 cu. ft., the second 6 cd. 6 cd. ft., and the third 9 cd. 4 cd. ft. 4 cu. ft.? *21 cd. 6 cd. ft.*

7. A father is 28 yr. 164 da. older than his son, and the son is 17 yr. 225 da. old. How old is the father? *46 yr. 24 da.*

187. *Rule for Addition of Compound Numbers.*

I. *Write the parts with like denominations in the same column.*

II. *Add each denomination separately, beginning with the lowest; and when the sum is less than 1 of the next higher denomination, write it under the denomination added.*

III. *When the sum of any denomination is equal to or more than 1 of the next higher denomination, reduce it to that higher denomination, write the remainder under the denomination added, and add the quotient with the next higher denomination.*

PROBLEMS.

8. A stationer sold 5 gro. 3 doz. 8 pens of one kind, and 2 gro. 6 doz. 6 pens of another. How many pens did he sell? *7 gro. 10 doz. 2.*

9. A livery-man bought 3 loads of hay, the first weighing 1 T. 2 cwt. 17 lb., the second 1 T. 3 cwt. 96 lb., and the third 19 cwt. 49 lb. How much hay did he buy? *3 T. 5 cwt. 62 lb.*

10. A housekeeper made 2 gal. 1 qt. 1 pt. of currant wine, 1 gal. 3 qt. of blackberry wine, and 4 gal. 2 qt. 1 pt. of grape wine. How much wine did she make? *8 gal. 3 qt.*

11. In five successive days, a fruit dealer sold 3 pk. 3 qt. 1 pt. of cherries, 1 bu. 1 pk. 5 qt., 1 bu. 3 pk. 7 qt., 1 bu. 2 qt. 1 pt., and 3 pk. 1 pt. How many cherries did he sell?

12. What is the sum of 5 rm. 14 quires 12 sheets, 7 rm. 11 quires 9 sheets, and 9 rm. 15 quires 9 sheets?

13. A railroad train runs from Detroit to Ann Arbor in 1 h. 45 min.; to Jackson in 1 h. 40 min. more; to Marshall in 1 h. 25 min. more; to Kalamazoo in 1 h. 35 min. more; to Niles in 2 h. 15 min. more; and to Chicago in 4 h. 30 min. more. What is the running time of the train from Detroit to Chicago? *13 h. 10 min.*

14. In an ax factory are six large grindstones, which weigh 2 T. 1 cwt. 18 lb., 1 T. 16 cwt. 24 lb., 2 T. 3 cwt. 7 lb., 2 T. 2 cwt. 7 lb., 1 T. 18 cwt. 87 lb., and 1 T. 19 cwt. 69 lb. What is their total weight? *12 T. 1 cwt. 12 lb.*

15. A telegraph company put up 1 mi. 14 rd. 3 yd. of wire one day, 318 rd. 5 yd. the second day, 1 mi. 39 rd. 4 yd. the third day, and 1 mi. 67 rd. the fourth day. How much wire did they put up in the four days? *4 mi. 120 rd. 1 yd.*

16. What is the sum of 9 cu. yd. 20 cu. ft. 388 cu. in., 218 cu. yd. 14 cu. ft. 524 cu. in., and 145 cu. yd., 11 cu. ft. 1415 cu. in.? *373 cu. yd. 19 cu. ft. 599 cu. in.*

17. There are 35 sq. yd. 5 sq. ft. of plastering in the ceiling of a room, 22 sq. yd. 2 sq. ft. in each of the two side walls, and 17 sq. yd. 7 sq. ft. in each of the two end walls. How much plastering in the room? *115 sq. yd. 5 sq. ft.*

18. Add 4 yd. 2 ft. 4 in., 3 yd. 1 ft. 8 in., 5 yd. 2 ft. 6 in.

19. Add 28 wk. 4 da. 14 h. 45 min. 45 sec., 11 wk. 3 da. 10 h. 30 min. 15 sec., and 6 wk. 6 da. 3 h. 25 min. 30 sec.

SECTION III.

SUBTRACTION.

188. Ex. 1. From 8 bu. 1 pk. 7 qt. subtract 4 bu. 3 pk. 2 qt.

EXPLANATION.—We write the denominations of the subtrahend under the like denominations of the minuend. Commencing with the lowest denomination, we subtract;

SOLUTION.

8 *bu.*	1 *pk.*	7 *qt.*
4	3	2
3 *bu.*	2 *pk.*	5 *qt.*

2 qt. from 7 qt. leave 5 qt., which we write as the quarts of the remainder. Since we can not subtract 3 pk. from 1 pk., we take 1 bu. (= 4 pk.) of the 8 bu. in the minuend, add it to the 1 pk., and from the 5 pk. thus obtained, we subtract the 3 pk., writing the difference, 2 pk., in the remainder. Finally, we subtract 4 bu. from the 7 bu. now left in the minuend, and write the difference, 3 bu., in the remainder. The compound number, 3 bu. 2 pk. 5 qt., is the remainder required.

Ex. 2. From 20 gal. subtract 5 gal. 2 qt. 1 pt.

EXPLANATION.—Since we can not subtract 1 pt. from 0 pt., and there are no quarts in the minuend to reduce to pints, we take 1 of the 20 gal., leaving 19 gal. From this 1 gal. (or 4 qt.) we take 1 qt., leaving 3 qt.; and this 1 qt. = 2 pt. The form of the minuend is now changed from 20 gal. to 19 gal. 3 qt. 2 pt., and from this we subtract 5 gal. 2 qt. 1 pt., obtaining a remainder of 14 gal. 1 qt. 1 pt.

SOLUTION.

19	3	2
20 *gal.*		
5	2 *qt.*	1 *pt.*
14 *gal.*	1 *qt.*	1 *pt.*

PROBLEMS.

(1)

From	7 *da.*	3 *h.*	20 *min.*
Subtract	3	9	15

(2)

5 *mi.*	220 *rd.*	4 *yd.*	2 *ft.*	5 *in.*
2	264	3	2	8

3. From a cask that contained 33 gal. 2 qt. of vinegar, a grocer drew 17 gal. 3 qt. How much vinegar was left in the cask? *15 gal. 3 qt.*

4. A farmer raised 614 bu. 1 pk. of oats, and sold all but 133 bu. 3 pk. How many oats did he sell? *480 bu. 2 pk.*

5. A physician bought a load of hay which weighed, with the wagon, 1 T. 8 cwt. 21 lb. The wagon alone weighed 12 cwt. 43 lb. What was the weight of the hay? *15 cwt. 78 lb.*

6. A merchant tailor bought 32 gro. 6 doz. rubber buttons, and sold 24 gro. 8 doz. 6. How many buttons had he left?

7. The walls of a room measure 68 sq. yd. 4 sq. ft., and the windows and doors 18 sq. yd. 7 sq. ft. How many square yards of plastering on the walls? *49 sq. yd. 6 sq. ft.*

8. A farmer exchanged a farm of 200 acres for another containing 113 A. 28 sq. rd. How much more land was there in the first farm than in the second? *86 A. 132 sq. rd.*

9. A bookseller having 22 rm. 12 quires of letter-paper, sold 13 rm. 16 quires 12 sheets. How much paper had he then? *8 rm. 15 quires 12 sheets.*

10. A grocer bought a crock of butter which weighed 44 lb. 6 oz. The crock alone weighed 7 lb. 10 oz. How much did the butter weigh? *36 lb. 12 oz.*

11. A druggist put 5 gal. 2 qt. 1 pt. of alcohol into a can which would hold 20 gal. How much more alcohol would the can have held? *14 gal. 1 qt. 1 pt.*

12. A laborer agreed to dig a cellar 18 ft. long, 16 ft. wide, and 5 ft. deep. After digging 44 cu. yd. of earth, how much more has he to remove? *9 cu. yd. 9 cu. ft.*

13. William is 16 yr. 28 da. old, and Edward is 11 yr. 284 da. old. How much older is William than Edward? *4 yr. 109 da.*

14. A farmer contracts to deliver at a railroad station 1,000 cords of wood. He has 384 cd. 5 cd. ft. already cut. How much more wood must he chop? *615 cd. 3 cd. ft.*

15. From 90 cu. yd. subtract 39 cu. yd. 18 cu. ft. 966 cu. in. *50 cu. yd. 8 cu. ft. 762 cu. in.*

Ex. 3. How many years, months, and days from April 15, 1864, to July 4, 1867?

SOLUTION.

1867	*yr.* 7	*mo.* 4	*da.*
1864	4	15	
	3 *yr.*	2 *mo.*	19 *da.*

EXPLANATION. — Since the later of two dates is expressed by a greater compound number than the earlier, we write the later date for the minuend, and the earlier for the subtrahend, writing the number of the year, month, and day in order. We then subtract as in other compound numbers, calling 30 days a month when the number of days in the subtrahend is greater than that in the minuend.

189. *Rule for Subtraction of Compound Numbers.*

I. *Write the denominations of the subtrahend under the like denominations of the minuend.*

II. *Subtract each denomination of the subtrahend from the like denomination of the minuend, and write the result as the same denomination in the remainder.*

III. *When any denomination of the subtrahend exceeds that in the minuend, before subtracting, reduce 1 of the next higher denomination of the minuend to this lower denomination, and add it to the number of this denomination given in the minuend.*

IV. *In the last case, in subtracting the next higher denomination, we may either call the number in the minuend 1 less, or that in the subtrahend 1 more.*

PROBLEMS.

16. Benjamin Franklin was born Jan. 17, 1706, and died Apr. 17, 1790. How old was he when he died? *84 yr. 3 mo.*

17. George Washington was born Feb. 22, 1732, and died Dec. 14, 1799. At what age did he die? *67 yr. 9 mo. 22 da.*

18. A note dated June 7, 1863, was paid Apr. 4, 1865. How long did it remain unpaid? *1 yr. 9 mo. 27 da.*

19. A note was given Sept. 10, 1867, payable Feb. 4, 1868. How long had it to run? *4 mo. 24 da.*

20. Robert was born Oct. 9, 1858. How old was he, May 11, 1867? *8 yr. 7 mo. 2 da.*

21. Washington Irving died Nov. 28, 1859, aged 76 yr. 7 mo. 25 da. What was the date of his birth? *Apr. 3, 1783.*

22. A farmer in a country district agrees to furnish 10 cd. 4 cd. ft. of wood for the winter term of school. After drawing 4 cd. 7 cd. ft., how much has he yet to draw? *5 cd. 5 cd. ft.*

23. From 11 mi. 84 rd. 4 yd. 1 ft. take 5 mi. 186 rd. 2 yd. 2 ft. *5 mi. 218 rd. 1 yd. 2 ft.*

24. In a storehouse there is a bin which will hold 240 bu., and in the bin are 183 bu. 3 pk. of wheat. How much more wheat will the bin hold? *56 bu. 1 pk.*

SECTION IV.

MULTIPLICATION.

190. Ex. Multiply 6 wk. 2 da. 8 h. by 7.

SOLUTION.

6 *wk.* 2 *da.* 8 *h.*
7
44 *wk.* 2 *da.* 8 *h.*

EXPLANATION.—We write the multiplier under the lowest denomination of the multiplicand. Then, commencing at the right, we multiply; 7 times 8 h. = 56 h. But 56 h. = 2 da. 8 h.; we therefore write the 8 h. in the product, and reserve the 2 da. to be added with the days of the product. 7 times 2 da. = 14 da., and 14 da. + 2 da. = 16 da. But since 16 da. = 2 wk. 2 da., we write the 2 da. in the product, reserving the 2 wk. to be added with the weeks in the product. 7 times 6 wk. = 42 wk., and 42 wk. + 2 wk. = 44 wk., which we write in the product. The result, 44 wk. 2 da. 8 h., is the product required.

PROBLEMS.

(1) Multiply 3 *T.* 4 *cwt.* 18 *lb.* by 6

(2) 1 *cu. yd.* 14 *cu. ft.* 356 *cu. in.* by 7

3. Multiply 4 bu. 2 pk. 7 qt. by 9.

4. If a painter uses 2 gal. 3 qt. 1 pt. of linseed-oil in painting 1 lumber wagon, how much will he use in painting 5 wagons? *14 gal. 1 qt. 1 pt.*

5. If a man can cradle an acre of wheat in 3 h. 20 min., how long will he be in cutting 7 acres? *23 h. 20 min.*

6. If the rate of speed of a railroad train is 28 mi. 216 rd. an hour, how far will it run in 14 hours? *401 mi. 144 rd.*

7. What is the weight of 50 bales of cotton, each weighing 4 cwt. 96 lb.? *12 T. 8 cwt.*

8. How many bushels of wheat will a field of 9 acres yield, at an average of 14 bu. 3 pk. 4 qt. an acre? *133 bu. 3 pk. 4 qt.*

9. How much wood can a team draw at 18 loads, if they draw 1 cd. 2 cd. ft. 12 cu. ft. at each load? *24 cd. 1 cd. ft. 8 cu. ft.*

191. *Rule for Multiplication of Compound Numbers.*

I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply each denomination of the multiplicand, in order, by the multiplier, as in integers; and when the product is less than 1 of the next higher denomination, write it under the denomination multiplied.*

III. *When any product is equal to, or greater than, 1 of the next higher denomination, reduce it to that higher denomination, write the remainder under the denomination multiplied, and add the quotient with the next higher denomination in the final result.*

PROBLEMS.

10. Multiply 12 A. 84 sq. rd. by 27. *338 A. 28 sq. rd.*

11. If a farmer uses 1 bu. 3 pk. 2 qt. of seed-wheat to the acre, how much will he use in seeding 15 acres? *27 bu. 6 qt.*

12. How much cider will it take to fill 8 demijohns, each holding 3 gal. 2 qt. 1 pt.? *29 gal.*

13. A publisher uses 2 rm 7 quires 12 sheets for each number of a weekly newspaper. How much paper does he use in a year? *123 rm. 10 quires.*

14. If the water of a river flows at the rate of 3 mi. 280 rd. an hour, how far will a log float in 219 hours? *848 mi. 200 rd.*

15. If 33 cd. 7 cd. ft. of wood make 1 canal-boat load, how much wood will make 19 loads? *643 cd. 5 cd. ft.*

16. A farm hand can plow an acre of corn in 4 h. 15 min. How long will it take him to plow 25 acres, if he works 10 hours a day? *10 da. 6 h. 15 min.*

17. How much land is there in 24 village lots, each 5 rods front and 7 rods deep? *5 A. 40 sq. rd.*

18. A teamster drew 32 loads of freight, each load weighing 1 T. 2 cwt. 25 lb. How much freight did he draw? *35 T. 12 cwt.*

19. If a manufacturer makes 15 gro. 4 doz. 9 clothes pins each day, how many does he make in the 308 working-days of the year? *4,741 gro. 11 doz.*

SECTION V.

DIVISION.

192. Ex. Divide 16 rm. 9 quires 14 sheets of paper into 5 equal parts.

SOLUTION.

16 *rm.* 9 *quires* 14 *sheets* ⌊ 5

3 *rm.* 5 *quires* 22 *sheets*

EXPLANATION.—We write the dividend and divisor, and commence at the left of the dividend to divide, as in integers. One fifth of 16 rm. is 3 rm. with a remainder of 1 rm. Writing the 3 rm. in the quotient, we reduce the 1 rm. remainder to quires, and to it add the 9 quires, making 29 quires. One fifth of 29 quires is 5 quires with a remainder of 4 quires. Writing the 5 quires in the quotient, we reduce the 4 quires remainder to sheets, and to it add the 14 sheets, making 110 sheets. One fifth of 110 sheets is 22 sheets, which we write in the quotient. The result, 3 rm. 5 quires 22 sheets, is the quotient required.

PROBLEMS.

Find the quotient in problems 1, 2, 3.

(1)	(2)	(3)
52 *bu.* 3 *pk.* 4 *qt.* ⌊ 9	300 *gro.* 4 *doz.* ⌊ 16	401 *cd.* 5 *cd. ft.* ⌊ 17

4. A ship sailed 59 mi. 20 rd. in 7 hours. What was her average hourly distance? *8 mi. 140 rd.*

5. A farmer put 385 gal. 3 qt. 1 pt. of cider into 9 casks. How much cider did he put into each? *42 gal. 3 qt. 1 pt.*

6. If a glazier can set the glass for 8 windows in 10 h. 40 min., how long will it take him to set the glass for 1 window?

7. A teamster fed to his horses 67 bu. 2 pk. of oats in 30 days. How many oats did he feed each day? *2 bu. 1 pk.*

8. If a locomotive burns 13 cd. 6 cd. ft. 4 cu. ft. of wood in making 12 trips, how much does it burn in making 1 trip?

193. *Rule for Division of Compound Numbers.*

I. *Write the dividend and divisor, as in integers.*

II. *Divide the highest given denomination, as in integers, and write the result as the corresponding denomination in the quotient.*

III. *Reduce the remainder to the next lower denomination, add to the result the number given of this lower denomination, and divide the same as before.*

IV. *Proceed in the same manner until all the denominations of the dividend are divided.*

PROBLEMS.

9. From 15 acres of meadow a farmer cut 28 T. 13 cwt. 75 lb. of hay. What was the yield per acre? *1 T. 18 cwt. 25 lb.*

10. He harvested 376 bu. 3 pk. 4 qt. of barley from 12 acres. What was the yield of barley per acre? *31 bu. 1 pk. 5 qt.*

11. A workman laid 64 rd. 3 yd. 1 ft. of stone-wall in 26 days. How much did he lay each day? *2 rd. 2 yd. 2 ft.*

12. How long will it take a cooper to make 1 flour barrel, if he can make 8 in 10 hours? *1 h. 15 min.*

13. A bridge pier containing 448 cu. yd. of stone was built in 36 days. What was the average amount of stone laid daily? *12 cu. yd. 12 cu. ft.*

14. Divide 69 T. 3 cwt. 29 lb. 8 oz., by 19.

Quotient, 3 T. 12 cwt. 80 lb. 8 oz.

15. A cook uses 12 doz. eggs in 18 days. How many eggs does she use each day? *8.*

16. An ink manufacturer put up 3 gal. 2 qt. 1 pt. 2 gi. of ink in 59 bottles. How much did each bottle contain? *2 gi.*

17. If 1 rm. 10 quires of paper are used in making 10 doz. writing-books, how many sheets are used in making 1 doz. books? How many in making 1 book? *6 sheets in 1 book.*

18. If 12 men can chop 132 cords of wood in 4 days, how many cords can 1 man chop in 1 day? *2 cd. 6 cd. ft.*

19. If 7 men can mow 26 A. 40 sq. rd. of grass in 10 hours, how much can 1 man mow in 1 hour? *60 sq. rd.*

SECTION VI.

PROBLEMS IN COMPOUND NUMBERS.

1. How many bales, each weighing 250 lb., will 7 T. 5 cwt. of hay make? *58.*

2. At $3 a rod, how much will it cost to build a fence around a lot 8 rods long and 5 rods wide? *$78.*

3. A man traveled by stage 78 miles in 16 hours. How far did he travel in an hour? *4 mi. 280 rd.*

4. How much will 3 T. 5 cwt. 56 lb. of iron castings cost, at 7 cents a pound? *$458.92.*

5. How many cu. yd. of stone are there in an abutment 56 ft. long, 8 ft. wide, and 15 ft. high? *248 cu. yd. 24 cu. ft.*

6. How many gallons of wine will 25 doz. quart bottles hold? *75.*

7. Five wood-cutters worked together during the winter. The first chopped 118 cd. 4 cd. ft., the second 109 cd. 2 cd. ft. 8 cu. ft., the third 106 cd. 5 cd. ft., the fourth 98 cd. 3 cd. ft. 8 cu. ft., and the fifth 91 cd. 7 cd. ft. How much wood did they chop? *524 cd. 6 cd. ft.*

8. From Dec. 30 to Aug. 1 of the following year, how many months and days? *7 mo. 2 da.*

9. How many minutes from Aug. 18, at noon, to Oct. 9, at noon? *74,880 min.*

10. How much cider can be made from 100 bu. of apples, if 3 gal. 1 qt. 1 pt. can be made from 1 bu.? *337 gal. 2 qt.*

11. From a barrel which contained 42 gal. 2 qt. of syrup, 7 gal. 2 qt. were drawn one day, 3 gal. 1 qt. 1 pt. the next, and 4 gal. 3 qt. the third? How much syrup remained in the barrel? *26 gal. 3 qt. 1 pt.*

12. A railroad company has a pile of wood 576 ft. long, 25 ft. wide, and 18 ft. high. How much wood is there in the pile? *2,025 cd.*

13. In 17 gal. 1 pt., how many gills? *548.*

14. At $.625 a cu. yd., how much will it cost to dig a cellar 27 ft. long, 19 ft. wide, and 7 ft. deep? *$83.125.*

15. A way-freight car was loaded with 3 T 3 cwt. 48 lb. of groceries, 3 T. 19 cwt. 40 lb. of hardware, 1 T. 1 cwt. 94 lb. of furniture, and 18 cwt. 64 lb. of dry goods. How much freight was in the car? *9 T. 3 cwt. 46 lb.*

16. A druggist bought 9 casks, each containing 20 gal. 3 qt. 1 pt. of brandy. How much did they all contain?

17. How much seed-corn will be required for 5,000 pint papers? *78 bu. 4 qt.*

18. How many cu. in. are 3 cu. yd. 18 cu. ft. 324 cu. in.?

19. If 2 show-bills can be printed on 1 sheet, how much paper will be required for 2,400 bills? *2 rm. 10 quires.*

20. At $35 an acre, what will be the cost of a piece of land 140 rd. long, and 112 rd. wide? *$3,430.*

21. How many 3-pint bottles will 32 gal. 2 qt. 1 pt. of cider fill? *87.*

22. If it is 18 feet around the hind wheel of a carriage, how many times will the wheel turn over in running 6 mi. 174 rd. 3 yd.? *1,920.*

23. A certain room is 22 ft. long, 18 ft. wide, and 12 ft. high. How many sq. yd. are there in the ceiling? *44.*

24. How many square yards are there in the four sides of the same room? *106 sq. yd. 6 sq. ft.*

25. A grocer bought 6 doz. brooms for $19.34, and retailed them at 40 cents apiece. How much did he gain?

26. In making a road 250 rods long, 5 cu. yd. 3 cu. ft. of gravel were used to the rod. How much gravel was used? *1,277 cu. yd. 21 cu. ft.*

27. If a manufacturer makes 1,000 lead-pencils in a day, how many gross will he make in 26 days? *180 gro. 6 doz. 8.*

28. A man bought 7 acres of land, at $450 an acre, and sold it in building lots, each 10 rd. long and 4 rd. wide, at $150 apiece. How much did he gain? *$1,050.*

29. In 1 liquid gallon there are 231 cu. in. What is the capacity in gallons of a cistern 7 ft. long, 5.5 ft. wide, and 9 ft. deep? *2,592.*

30. How long will 3 cwt. of sugar last a family, if they use 1 lb. a day? *42 wk. 6 da.*

31. What will be the cost of a pile of stone 30 ft. long, 8 ft wide, and 4 ft. high, at $6 a cord? *$45.*

32. How many miles will a locomotive run in 4 hours, running at the rate of 124 rods in a minute? *93.*

33. Last year, I sold from my garden 5 bu. 1 qt. of cherries, dried 1 bu. 3 pk. 1 qt., put up in cans 3 pk. 5 qt., and 3 pk. 2 qt. were eaten in my family. How many cherries grew in my garden? *8 bu. 2 pk. 1 qt.*

34. These cherries grew upon 7 trees. What was the average yield per tree? *1 bu. 7 qt.*

35. How many tons of hay will a span of horses eat in 15 weeks, if they eat 45 lb. a day? *2 T. 7 cwt. 25 lb.*

36. How many oats will it take to last them the same time, if they eat 24 qt. a day? *78 bu. 3 pk.*

37. If a family use 3 bu. 1 pk. of potatoes each month, how much will a year's supply cost them, at $.5625 a bushel?

38. If a housekeeper uses a half-pint of molasses each day, how long will 20 gallons last her? *45 wk. 5 da.*

39. A plumber has a coil of lead pipe 34 ft. long, which weighs 189 lb. 2 oz. How much does 1 ft. of the pipe weigh?

40. How much will 15 ft. of the same pipe weigh?

41. If I deposit $4.50 in a savings-bank every week, and draw out $12.50 each month, how much will I have on deposit at the end of the year? *$84.*

42. How many cords of stone will it take to build a stone fence 76 ft. long, 4 ft. high, and 2 ft. thick? *4.75 cd.*

43. One day a carman drew 14 T. 18 cwt. 52 lb. of freight, at 17 equal loads. How much did he draw at each load?

44. A grocer bought four hogsheads of molasses, which contained 118 gal. 3 qt., 123 gal. 2 qt., 109 gal., and 122 gal. 1 qt. How much molasses did he buy?

45. From the first hogshead he sold 49 gal., from the second 68 gal. 3 qt., from the third 39 gal. 1 qt., and from the fourth 54 gal. 2 qt. How much molasses did he sell?

46. How much molasses was left in each hogshead?
In 1st and 3d, 69 gal. 3 qt.; in 2d, 54 gal. 3 qt.; in 4th, 67 gal. 3 qt.

47. How much molasses had he on hand? *262 gal.*

CHAPTER IV.

FRACTIONS.

SECTION I.

INDUCTION AND NOTATION.

194. When an apple is divided into 2 equal parts, 1 of the parts is 1 half. When a pear or any thing is divided into 3 equal parts, 1 of the parts is 1 third, and 2 of the parts are 2 thirds. When a thing is divided into 4 equal parts, the parts are fourths.

1 half is written $\frac{1}{2}$; 1 third, $\frac{1}{3}$; 2 thirds, $\frac{2}{3}$; 1 fourth, $\frac{1}{4}$; 2 fourths, $\frac{2}{4}$; 3 fourths, $\frac{3}{4}$.

When a thing or a number is divided into 5 equal parts, the parts are fifths; when into 6 equal parts, they are sixths; when into 7 equal parts, they are sevenths; and when into 8 equal parts, they are eighths. Fifths, sixths, sevenths, and eighths are written thus:

1 fifth, $\frac{1}{5}$,	2 sixths, $\frac{2}{6}$,	1 seventh, $\frac{1}{7}$,	4 eighths, $\frac{4}{8}$,
2 fifths, $\frac{2}{5}$,	4 sixths, $\frac{4}{6}$,	3 sevenths, $\frac{3}{7}$,	5 eighths, $\frac{5}{8}$,
4 fifths, $\frac{4}{5}$.	5 sixths, $\frac{5}{6}$.	6 sevenths, $\frac{6}{7}$.	7 eighths, $\frac{7}{8}$.

195. A number that represents one or more of the equal parts into which a thing is divided is a *Fraction.*

196. The two numbers that are used in writing a fraction are the *Terms.* Thus the terms of the fraction $\frac{5}{6}$ are 5 and 6. The 6 shows that a whole thing is divided into 6 equal parts, and the 5 represents 5 of these parts.

197. The term that expresses the number of equal parts into which a whole one is divided is written below a horizontal line, and is the *Denominator;* and

198. The term that represents the number of these equal parts is written above the line, and is the *Numerator.* Thus, in the fraction $\frac{4}{7}$, 4 and 7 are the terms; 4 is the numerator, and 7 is the denominator.

199. When the numerator is less than the denominator, the fraction is less than 1; as, $\frac{2}{3}$, $\frac{5}{7}$, $\frac{1}{8}$, $\frac{5}{12}$, $\frac{7}{15}$.

When the numerator and denominator are equal, the value of the fraction is 1; as, $\frac{5}{5}=1$, $\frac{8}{8}=1$, $\frac{12}{12}=1$, $\frac{15}{15}=1$, $\frac{24}{24}=1$.

When the numerator is greater than the denominator, the value of the fraction is greater than 1; as, $\frac{7}{2}$, $\frac{4}{3}$, $\frac{12}{5}$, $\frac{10}{9}$, $\frac{15}{12}$, $\frac{27}{20}$.

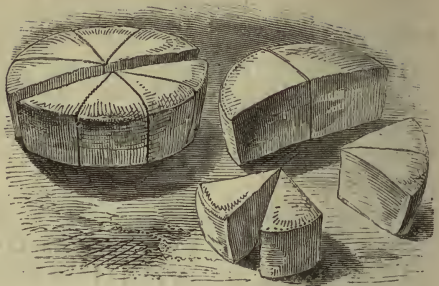
200. A fraction whose value is less than 1 is a *Proper Fraction;* as, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{4}{11}$, $\frac{19}{20}$, $\frac{5}{37}$.

201. A fraction whose value is equal to or greater than 1 is an *Improper Fraction;* as, $\frac{4}{4}$, $\frac{9}{9}$, $\frac{6}{4}$, $\frac{11}{7}$, $\frac{12}{5}$, $\frac{16}{15}$.

202. Fractions whose denominators are alike are *Similar Fractions;* as, $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$; $\frac{2}{9}$, $\frac{5}{9}$, $\frac{1}{9}$, and $\frac{8}{9}$.

203. Fractions whose denominators are unlike are *Dissimilar Fractions;* as, $\frac{4}{5}$, $\frac{3}{9}$.

204. A number composed of an integer and a fraction is a *Mixed Number;* as, $4\frac{2}{3}$, $31\frac{1}{4}$. (See **119.**)



205. Two of the 8 equal parts of this cake, or $\frac{2}{8}$, are 2 times as much as 1 of the parts, or $\frac{1}{8}$; and $\frac{4}{8}$ are 2 times as much as $\frac{2}{8}$. Hence, $2 \times \frac{1}{8} = \frac{2}{8}$, and $2 \times \frac{2}{8} = \frac{4}{8}$. So, also, $3 \times \frac{1}{8} = \frac{3}{8}$, $3 \times \frac{2}{5} = \frac{6}{5}$, and $4 \times \frac{2}{9} = \frac{8}{9}$.

If we divide $\frac{2}{8}$ of the cake into 2 equal parts, 1 part will be $\frac{1}{8}$; if we divide $\frac{4}{8}$ of it into 2 equal parts, each of the 2 equal parts will be $\frac{2}{8}$. So, also, $\frac{4}{5} \div 2 = \frac{2}{5}$, $\frac{6}{7} \div 2 = \frac{3}{7}$, $\frac{2}{3} \div 2 = \frac{1}{3}$, $\frac{8}{11} \div 2 = \frac{4}{11}$. Hence,

I. *A fraction may be multiplied by multiplying its numerator.*

II. *A fraction may be divided by dividing its numerator.*

206. If we divide a whole cake into 2 equal parts, each part will be $\frac{1}{2}$; if we divide $\frac{1}{2}$ of it into 2 equal parts, each part will be $\frac{1}{4}$; and if we divide $\frac{1}{4}$ of it into 2 equal parts, each part will be $\frac{1}{8}$. That is, $1 \div 2 = \frac{1}{2}$, $\frac{1}{2} \div 2 = \frac{1}{4}$, and $\frac{1}{4} \div 2 = \frac{1}{8}$.

Again, 2 of the eighths put together are $\frac{1}{4}$, 2 of the fourths together are $\frac{1}{2}$, and the 2 halves are the whole cake, or 1. That is, $2 \times \frac{1}{8} = \frac{1}{4}$, $2 \times \frac{1}{4} = \frac{1}{2}$, and $2 \times \frac{1}{2} = 1$.

I. *A fraction may be divided by multiplying its denominator.*

II. *A fraction may be multiplied by dividing its denominator.*

207. If we divide $\frac{1}{2}$ of a melon into 2 equal parts, we shall have $\frac{2}{4}$ of a melon; and if we divide $\frac{1}{4}$ of a melon

into 2 equal parts, we shall have $\frac{2}{8}$ of a melon. That is, $\frac{1}{2} = \frac{2}{4}$, and $\frac{1}{4} = \frac{2}{8}$. But the $\frac{1}{2}$ may be changed to $\frac{2}{4}$, and the $\frac{1}{4}$ to $\frac{2}{8}$, by multiplying both terms of each fraction by 2. Thus, $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$, and $\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$.

Again, $\frac{2}{8}$ of the melon are together equal to $\frac{1}{4}$ of it, and $\frac{2}{4}$ of a melon are together equal to $\frac{1}{2}$ of it. That is, $\frac{2}{8} = \frac{1}{4}$, and $\frac{2}{4} = \frac{1}{2}$. But the $\frac{2}{8}$ may be changed to $\frac{1}{4}$, and the $\frac{2}{4}$ to $\frac{1}{2}$, by dividing both terms of each fraction by 2. Thus, $\frac{2}{8} \div 2 = \frac{1}{4}$, and $\frac{2}{4} \div 2 = \frac{1}{2}$. Hence,

I. *The value of a fraction is not changed by multiplying both terms by the same number.*

II. *The value of the fraction is not changed by dividing both terms by the same number.*

208. All operations in fractions are based upon the following

General Principles of Fractions.

I. *A fraction is multiplied,*
   1. *By multiplying its numerator;* or,
   2. *By dividing its denominator.*

II. *A fraction is divided,*
   1. *By dividing its numerator;* or,
   2. *By multiplying its denominator.*

III. *The value of a fraction is not changed,*
   1. *By multiplying both terms by the same number;* or,
   2. *By dividing both terms by the same number.*

(See Manual, page 219.)

SECTION II.

REDUCTION.

CASE I.

Fractions to Lowest Terms.

209. When the terms of a fraction can not both be exactly divided by any integer greater than 1, the fraction is in its lowest terms. Thus, $\frac{5}{7}$ is in its lowest terms, because no integer greater than 1 will exactly divide both 5 and 7.

210. To reduce a fraction to its lowest terms is to change its numerator and denominator to the smallest numbers possible, without changing the value of the fraction. Thus, $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$.

Ex. Reduce $\frac{12}{18}$ to its lowest terms.

EXPLANATION.—We reduce $\frac{12}{18}$ to lower terms by dividing both numerator and denominator by 2, $(\frac{12}{18} = \frac{6}{9})$; and the fraction, $\frac{6}{9}$, thus obtained, we reduce to still lower terms by dividing both terms by 3, $(\frac{6}{9} = \frac{2}{3},)$ as shown in the First Solution. (See Prin. III., 2.) Or we can reduce $\frac{12}{18}$ to its lowest terms at one operation by dividing both terms by 6, as shown in the Second Solution.

FIRST SOLUTION.

$\frac{12}{18} = \frac{6}{9} = \frac{2}{3}$

SECOND SOLUTION.

$\frac{12}{18} = \frac{2}{3}$

PROBLEMS.

1. Reduce the fraction $\frac{6}{8}$ to its lowest terms. $\frac{3}{4}$.
2. Reduce $\frac{9}{15}$ mi. to its lowest terms. $\frac{3}{5}$ *mi.*
3. To what lower terms can $\frac{16}{24}$ be reduced? $\frac{8}{12}$, $\frac{4}{6}$, *or* $\frac{2}{3}$.
4. Reduce $\frac{8}{20}$, $\frac{15}{25}$, and $\frac{7}{35}$ to their lowest terms. $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{5}$.
5. In what lower terms can $\$\frac{81}{108}$ be expressed? $\$\frac{27}{36}$, $\$\frac{9}{12}$, *or* $\$\frac{3}{4}$.

6. Reduce $\frac{27}{63}$, $\frac{14}{16}$, and $\frac{49}{28}$ to their lowest terms.

7. Reduce the fractions $\frac{56}{63}$ and $\frac{121}{88}$ to their lowest terms.

$\frac{8}{9}$, $\frac{11}{8}$.

8. What are the lowest terms of the fractions $\frac{78}{102}$, $\frac{8}{18}$, $\frac{40}{23}$, $\frac{81}{36}$, $\frac{13}{143}$, and $\frac{105}{175}$? $\frac{13}{17}$, $\frac{4}{9}$, $\frac{8}{3}$, $\frac{9}{4}$, $\frac{7}{11}$, $\frac{3}{5}$.

CASE II.

Fractions to Given Denominators.

211. We have already seen that the value of a fraction is not changed by multiplying both terms by the same integer. Thus, $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$, $\frac{2}{4} \times \frac{3}{3} = \frac{6}{12}$, $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$, $\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$, and so on.

Ex. Reduce $\frac{2}{9}$ to thirty-sixths.

EXPLANATION.—To reduce 2 ninths to thirty-sixths, we must multiply both terms by such an integer as will give 36 for the new denominator. We find this integer by dividing 36 by 9. Multiplying both terms of $\frac{2}{9}$ by 4, the integer thus found, we have $\frac{8}{36}$.

SOLUTION.

$36 \mid 9$

4

$\frac{2}{9} \times \frac{4}{4} = \frac{8}{36}$

Hence, $\frac{2}{9} = \frac{8}{36}$.

PROBLEMS.

9. Reduce $\frac{3}{4}$ to twelfths. $\frac{9}{12}$.
10. Reduce $\frac{5}{7}$ to twenty-firsts.
11. Reduce $\frac{1}{6}$ to eighteenths, and $\frac{3}{8}$ to twenty-fourths. $\frac{3}{18}$, $\frac{9}{24}$.
12. Reduce $\frac{2}{3}$ to sixths, to ninths, and to fifteenths.

$\frac{4}{6}$, $\frac{6}{9}$, *and* $\frac{10}{15}$.

13. Reduce $\frac{5}{8}$ to forty-fifths, and $\frac{4}{11}$ to thirty-thirds.
14. Reduce $\frac{6}{7}$ to fourteenths and to twenty-firsts.
15. Reduce $\frac{4}{5}$ to fortieths and $\frac{5}{8}$ to fortieths. $\frac{32}{40}$, $\frac{25}{40}$.
16. Reduce $\frac{3}{4}$ and $\frac{6}{7}$ to twenty-eighths.

CASE III.

Dissimilar Fractions to Similar Fractions.

212. Ex. 1. Reduce $\frac{2}{3}$ and $\frac{1}{2}$ to similar fractions.

EXPLANATION.—Thirds can not be reduced to halves, nor halves to thirds. But since 2 times 3 = 6, we reduce $\frac{2}{3}$ to sixths by multiplying both terms by 2; and since 3 times 2 = 6, we reduce $\frac{1}{2}$ to sixths by multiplying both terms by 3.

SOLUTION.

$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$$

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

Hence, $\frac{2}{3}, \frac{1}{2} = \frac{4}{6}, \frac{3}{6}$.

Ex. 2. Reduce $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{7}$ to similar fractions.

EXPLANATION.—Since $3 \times 4 \times 7 = 84$, we reduce these fractions to 84ths by multiplying both terms of the first by 4 and 7, both terms of the second by 3 and 7, and both terms of the third by 3 and 4. Hence, $\frac{2}{3}, \frac{1}{4}, \frac{5}{7} = \frac{56}{84}, \frac{21}{84}, \frac{60}{84}$.

SOLUTION.

$$\frac{2}{3} \times \frac{4}{4} \times \frac{7}{7} = \frac{56}{84}$$

$$\frac{1}{4} \times \frac{3}{3} \times \frac{7}{7} = \frac{21}{84}$$

$$\frac{5}{7} \times \frac{3}{3} \times \frac{4}{4} = \frac{60}{84}$$

213. Hence, to reduce dissimilar to similar fractions, we

Multiply both terms of each fraction by the denominators of all the other fractions.

214. Fractions having like denominators are said to have a *common denominator;* and reducing dissimilar to similar fractions is sometimes called reducing them to equivalent fractions having a common denominator.

PROBLEMS.

17. Reduce $\frac{3}{4}$ and $\frac{1}{5}$ to similar fractions. $\frac{15}{20}, \frac{4}{20}$.
18. Reduce $\frac{1}{3}$ and $\frac{2}{7}$ to similar fractions.
19. Reduce $\frac{4}{5}$ and $\frac{1}{6}$ to similar fractions. $\frac{24}{30}, \frac{5}{30}$.
20. What similar fractions are equal to $\frac{3}{5}$ and $\frac{3}{8}$?
21. What similar fractions are equal to $\frac{2}{3}$ and $\frac{3}{10}$?
22. What similar fractions are equal to $\frac{13}{15}$ and $\frac{7}{8}$?

23. Reduce $\frac{5}{6}$ and $\frac{7}{11}$ to similar fractions. $\frac{55}{66}$, $\frac{42}{66}$.

24. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$ to similar fractions. $\frac{15}{30}$, $\frac{20}{30}$, $\frac{18}{30}$.

25. Reduce $\frac{3}{4}$, $\frac{1}{5}$, and $\frac{5}{9}$ to similar fractions. $\frac{135}{180}$, $\frac{36}{180}$, $\frac{100}{180}$.

26. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to equivalent fractions having a common denominator. $\frac{60}{120}$, $\frac{72}{120}$, $\frac{40}{120}$, $\frac{90}{120}$.

27. Reduce $\frac{5}{8}$, $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{2}{3}$ to similar fractions.

28. Reduce $\frac{1}{4}$, $\frac{5}{12}$, and $\frac{2}{7}$ to similar fractions.

29. What similar fractions are equal to $\frac{3}{8}$, $\frac{1}{3}$, and $\frac{2}{9}$?

30. Reduce $\frac{7}{10}$, $\frac{1}{5}$, $\frac{4}{5}$, and $\frac{4}{11}$ to similar fractions.

31. Reduce $\frac{1}{7}$, $\frac{1}{2}$, and $\frac{2}{3}$ to equivalent fractions having a common denominator. $\frac{6}{42}$, $\frac{21}{42}$, $\frac{28}{42}$.

CASE IV.

Improper Fractions to Integers or Mixed Numbers.

15. Ex. 1. In $\frac{12}{4}$ how many ones?

EXPLANATION.—Since every 4 fourths are 1, the number of times 12 fourths contains 4 fourths is the number of 1's in 12 fourths, and 12 fourths contains 4 fourths 3 times.

SOLUTION.

12 *fourths* ⌊ 4 *fourths*

3

Hence, $\frac{12}{4} = 3$.

Ex. 2. In $\frac{10}{3}$ how many ones?

EXPLANATION.—10 thirds contains 3 thirds 3 times, with a remainder of 1 third. (See Manual, p. 219.) Hence, $\frac{10}{3} = 3\frac{1}{3}$.

SOLUTION.

10 *thirds* ⌊ 3 *thirds*

$3\frac{1}{3}$

216. Hence, to reduce an improper fraction to an integer or a mixed number, we

Divide the numerator by the denominator.

PROBLEMS.

32. In $\frac{39}{3}$ how many 1's? *13.*

33. $\frac{136}{8}$ are how many 1's?

34. $\frac{95}{5}$ apples are how many apples?

35. $\frac{15}{4}$ melons are how many melons? $3\frac{3}{4}$.

36. Reduce $\frac{54}{4}$ to a mixed number. *$13\frac{2}{4}$, or $13\frac{1}{2}$.*

37. Reduce $\frac{15}{6}$, $\frac{27}{4}$, and $\frac{108}{25}$ to mixed numbers.

38. $\frac{143}{16}$ gallons are how many gallons? *$8\frac{15}{16}$.*

39. How much hay in 53 bales, each containing $\frac{1}{8}$ T.?

40. How many bushels of peaches in 176 baskets, each containing $\frac{1}{3}$ of a bushel? *$58\frac{2}{3}$.*

41. Reduce $\frac{387}{54}$, $\frac{5375}{23}$, and $\frac{34404}{148}$ to integers or mixed numbers. *$7\frac{1}{6}$, $233\frac{16}{23}$, 23.*

CASE V.

Integers or Mixed Numbers to Improper Fractions.

217. Ex. 1. Reduce 5 to fourths.

SOLUTION.

4 *fourths*
5
20 *fourths*

EXPLANATION. — Since 1 is 4 fourths, 5 are 5 times 4 fourths, or 20 fourths.

Hence, $5 = \frac{20}{4}$.

Ex. 2. Reduce $5\frac{3}{4}$ to an improper fraction.

FULL SOLUTION.	COMMON SOLUTION.
4 *fourths*	$5\frac{3}{4}$
5	4
20 *fourths*	20 + 3 = 23
3 *fourths*	
23 *fourths*	

EXPLANATION.—Since 1 is 4 fourths, 5 are 5 times 4 fourths, or 20 fourths, and 20 fourths + 3 fourths are 23 fourths.

Hence, $5\frac{3}{4} = \frac{23}{4}$.

(See Manual, page 219.)

218. Hence, to reduce an integer or a mixed number to an improper fraction, we

First multiply the integer by the denominator, and, if there be a numerator, add it to the product; then write this result for the numerator, and the given denominator for the denominator of the required fraction.

PROBLEMS.

42. In 15 how many thirds? *45.*

43. In $24\frac{1}{4}$ how many fourths? *$\frac{97}{4}$.*

44. What improper fraction is equal to $17\frac{1}{8}$? *$\frac{137}{8}$.*

45. Reduce $31\frac{5}{8}$ to an improper fraction. $\frac{2\,8\,4}{9}$.
46. Reduce 37 to sixths, and $11\frac{4}{15}$ to fifteenths.
47. What improper fraction is equal to $4\frac{3}{20}$?
48. How many fifty-seconds in $7\frac{11}{52}$? $\frac{3\,7\,5}{5\,2}$.
49. $12\frac{6}{13}$ equals how many thirteenths? $\frac{1\,6\,2}{1\,3}$.
50. $41 =$ how many thirty-firsts? $\frac{1\,2\,7\,1}{3\,1}$.
51. Reduce $24\frac{3}{8}$, and $2\frac{8}{2\,5}$ to improper fractions.
52. Reduce $5\frac{3\,6}{4\,5}$, $21\frac{4\,7}{6\,3}$, and $115\frac{1\,8}{2\,3\,7}$ to improper fractions. $\frac{2\,8\,1}{4\,9}$, $\frac{1\,3\,7\,0}{6\,3}$, $\frac{2\,7\,2\,7\,3}{2\,3\,7}$.

SECTION III.

ADDITION.

CASE I.

All the Parts Fractions.

219. Ex. 1. What is the sum of $\frac{1}{12}+\frac{3}{12}+\frac{5}{12}$ and $\frac{2}{12}$?

EXPLANATION.—Since the parts in these fractions are all of the same kind or denomination, (twelfths), and since the numerators express the numbers of the parts, we add the fractions by adding their numerators. $1 + 3 + 5 + 2 = 11$; and since the parts are twelfths, we write the denominator 12 under the 11.

SOLUTION.

$$\frac{1}{12}+\frac{3}{12}+\frac{5}{12}+\frac{2}{12}=\frac{11}{12}$$

Ex. 2. What is the sum of $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$?

EXPLANATION. — Fifths, thirds, and fourths do not express the same kind of things or parts, and hence they can not be directly added (See 20, I). But they can all be reduced to similar fractions, (sixtieths), and these similar parts can be added, as shown in Ex. 1.

FIRST SOLUTION.

$$\frac{4}{5}+\frac{1}{3}+\frac{3}{4}=\frac{48}{60}+\frac{20}{60}+\frac{45}{60}=\frac{113}{60}=1\frac{53}{60}$$

SECOND SOLUTION.

$$\frac{4}{5}+\frac{1}{3}+\frac{3}{4}=\frac{48+20+45}{60}=\frac{113}{60}=1\frac{53}{60}$$

In reducing the dissimilar fractions to similar ones, the common denominator need be written but once, and the several numerators may be written above it, as shown in the Second Solution. (See Manual, page 219.)

220. Hence, to add fractions, we

Reduce all dissimilar to similar fractions, add the numerators, and under the sum write the common denominator.

PROBLEMS.

1. What is the sum of $\frac{3}{4}$ and $\frac{1}{7}$? $\frac{25}{28}$.

2. Add $\frac{2}{5}$ and $\frac{3}{7}$.

3. Add $\frac{3}{8}$ and $\frac{4}{9}$. Add $\frac{3}{10}$ and $\frac{5}{13}$. $\frac{59}{72}$; $\frac{89}{130}$.

4. If a family burn $\frac{4}{7}$ T. of coal one month, and $\frac{3}{5}$ T. the next, how much do they burn in the two months? $\frac{41}{35}$, *or* $1\frac{6}{35}$.

5. What is the sum of $\frac{6}{7}$, $\frac{1}{5}$, and $\frac{1}{4}$? $\frac{183}{140}$, *or* $1\frac{43}{140}$.

6. A merchant sold $\frac{3}{4}$ bu. of clover seed to one farmer, $\frac{1}{2}$ bu. to another, and $\frac{5}{8}$ bu. to a third. How much clover seed did he sell? $\frac{120}{64}$, *or* $1\frac{7}{8}$ *bu.*

7. A teamster drew in three loads, $\frac{7}{8}$ cd. of wood, $\frac{15}{16}$ cd., and $\frac{11}{12}$ cd. How many cords of wood did he draw in the three loads? $\frac{449}{576}$, *or* $\frac{235}{48}$ *cd.*

8. What is the sum of $\frac{5}{9}$ and $\frac{3}{5}$?

9. Add $\frac{7}{10}$, $\frac{2}{3}$, and $\frac{3}{4}$. $\frac{254}{120}$, *or* $2\frac{7}{60}$.

10. A market gardener has $\frac{3}{4}$ of an acre of blackberries, $\frac{1}{3}$ of an acre of raspberries, and $\frac{3}{5}$ of an acre of strawberries. How many acres of berries has he?

11. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$ to twelfths, and find their sum.

12. Reduce $\frac{1}{2}$ da. $\frac{1}{4}$ da. $\frac{1}{6}$ da. and $\frac{1}{8}$ da. to twenty-fourths of a day, and add them. $\frac{25}{24}$, *or* $1\frac{1}{24}$ *days.*

13. Reduce $\$\frac{1}{4}$, $\$\frac{3}{5}$, $\$\frac{7}{10}$, and $\$\frac{1}{20}$ to hundredths of a dollar, and find their sum. $\$1\frac{60}{100}$.

14. A sewing girl paid $\$\frac{1}{2}$ for a thimble, $\$\frac{1}{10}$ for needles, $\$\frac{2}{5}$ for silk braid, $\$\frac{31}{100}$ for sewing silk, and $\$\frac{3}{20}$ for a spool of cotton. How many hundredths of a dollar did each article cost? How much money did she pay out? $\$1\frac{46}{100}$.

CASE II.

All or Some of the Parts Mixed Numbers.

221. Ex. What is the sum of $2\frac{4}{5}$, $5\frac{1}{2}$, 7, and $4\frac{2}{3}$?

EXPLANATION.—We write the parts with the integers in the same column, and reduce the dissimilar fractions to similar ones. Fifths, halves, and thirds can be reduced to thirtieths. Since $\frac{4}{5} = \frac{24}{30}$, $2\frac{4}{5}$ must equal $2\frac{24}{30}$. So, also, $5\frac{1}{2} = 5\frac{15}{30}$, and $4\frac{2}{3} = 4\frac{20}{30}$. Adding the fractions, we have $\frac{59}{30}$, or $1\frac{29}{30}$. We write the $\frac{29}{30}$ in the result, and add the 1 with the given integers. The sum of all the integers, 19, written before the $\frac{29}{30}$, gives $19\frac{29}{30}$, the required sum.

SOLUTION.

$$2\tfrac{4}{5} = 2\tfrac{24}{30}$$
$$5\tfrac{1}{2} = 5\tfrac{15}{30}$$
$$7 = 7$$
$$4\tfrac{2}{3} = 4\tfrac{20}{30}$$
$$19\tfrac{29}{30}$$

PROBLEMS.

15. Add $2\frac{2}{3}$ and $4\frac{4}{5}$. *$7\frac{7}{15}$.*

16. What is the sum of $5\frac{7}{9}$ and $3\frac{4}{7}$?

17. A copper-smith used $5\frac{3}{4}$ bu. of charcoal one month, and $6\frac{7}{8}$ bu. the next. How many bushels did he use in the two months? (Fourths may be reduced to eighths.) *$12\frac{5}{8}$ bu.*

18. A farmer has $85\frac{4}{5}$ A. of cleared land, and $47\frac{3}{4}$ A. of woodland. How many acres are in his farm?

19. What is the sum of $241\frac{5}{12}$, $4\frac{2}{3}$, and $1\frac{3}{5}$? *$247\frac{41}{60}$.*

20. Add $8\frac{5}{8}$, $6\frac{7}{9}$, and $27\frac{2}{3}$.

21. Mr. Wood's farm is $1\frac{3}{8}$ mi. long, and $\frac{3}{5}$ mi. wide. What is the distance around it? *$3\frac{49}{20}$ mi.*

22. A teamster drew two loads of straw, one weighing $1\frac{3}{20}$ T., and the other $\frac{37}{40}$ T. How much straw in both loads?

23. What is the sum of $\$4\frac{4}{5}$, $\$6\frac{7}{8}$, \$23, and $\$\frac{19}{20}$?

24. What is the sum of $1\frac{9}{16}$, $3\frac{1}{9}$, $5\frac{4}{7}$, and $10\frac{2}{3}$? *$21\frac{457}{630}$.*

25. A fruit dealer bought $1\frac{13}{32}$ bu. of walnuts of one boy, and $2\frac{3}{8}$ bu. of another. How many walnuts did he buy? *$3\frac{25}{32}$ bu.*

26. Add $3\frac{4}{7}$, 14, $5\frac{3}{13}$, $\frac{1}{5}$, $4\frac{1}{2}$. *$27\frac{457}{910}$.*

27. Add $19\frac{3}{4}$, $\frac{7}{15}$, $\frac{2}{9}$, $65\frac{4}{5}$, and 23. *$109\frac{43}{180}$.*

SECTION IV.

SUBTRACTION.

CASE I.

Both Numbers Fractions.

222. Ex. 1. From $\frac{11}{15}$ subtract $\frac{7}{15}$.

SOLUTION.

$$\frac{11}{15} - \frac{7}{15} = \frac{4}{15}$$

EXPLANATION. — Since the fractions are similar, we subtract the numerator of the less fraction from that of the greater. $11 - 7 = 4$; and since the parts are all of the same kind or denomination, (fifteenths), we write the denominator, 15, under the 4.

Ex. 2. From $\frac{4}{5}$ subtract $\frac{2}{3}$.

FIRST SOLUTION.

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

SECOND SOLUTION.

$$\frac{4}{5} - \frac{2}{3} = \frac{12 - 10}{15} = \frac{2}{15}$$

EXPLANATION.—Since fifths and thirds do not express the same kind of things or parts, (See 33, I), we reduce the given fractions to similar fractions, (fifteenths), and then subtract the less from the greater in the same manner as shown in Ex. 1.

In reducing the dissimilar fractions to similar ones, the common denominator need be written but once.

223. Hence, to subtract fractions, we

Reduce all dissimilar to similar fractions, subtract the less numerator from the greater, and under the difference write the common denominator.

PROBLEMS.

1. From $\frac{1}{3}$ subtract $\frac{3}{10}$. $\frac{1}{30}$.
2. From $\frac{5}{7}$ subtract $\frac{4}{9}$. $\frac{17}{63}$.
3. What is the difference between $\frac{13}{15}$ and $\frac{1}{2}$?
4. What is the difference between $\$\frac{5}{8}$ and $\$\frac{5}{10}$? $\$\frac{1}{8}$.

5. From a jug that contained $\frac{3}{4}$ of a gallon of boiled cider, a woman used $\frac{1}{8}$ of a gallon. How much cider was left in the jug? $\frac{5}{8}$ *gal.*

6. One day *A* worked $\frac{7}{10}$ of the day, and *B* $\frac{3}{4}$ of the day. Which worked the longer? How much the longer?

7. A housekeeper bought $\frac{1}{2}$ pk. of cranberries, and used $\frac{2}{5}$ pk. the same day. How many had she left? $\frac{1}{10}$ *pk.*

8. From $\frac{4}{7}$ subtract $\frac{4}{9}$. From $\frac{1}{3}$ subtract $\frac{2}{13}$

9. Charles lives $\frac{11}{16}$ mi. from the schoolhouse, and John $\frac{3}{5}$ mi. Which lives the greater distance from the schoolhouse? How much the greater? *Charles,* $\frac{7}{80}$ *mi.*

10. The snow was $\frac{7}{24}$ ft. deep one night, and the next morning it was $\frac{5}{8}$ ft. deep. What depth of snow had fallen during the night? $\frac{4}{3}$ *ft.*

11. From $\frac{7}{11}$ subtract $\frac{2}{9}$.

12. From $\frac{5}{14}$ subtract $\frac{1}{6}$. $\frac{4}{21}$.

13. If I pour $\frac{7}{16}$ qt. of wine from a bottle containing $\frac{7}{8}$ qt., how much wine will be left in the bottle? $\frac{7}{16}$ *qt.*

14. From $\frac{27}{32}$ cd. of wood a teamster took $\frac{7}{10}$ cd. How much wood remained?

15. A farmer bought $\frac{17}{20}$ T. of plaster, and sowed $\frac{7}{16}$ T. on his clover lot. What part of a ton had he left? $\frac{33}{80}$ *T.*

16. What is the difference between $\frac{13}{16}$ mi. and $\frac{3}{5}$ mi.? $\frac{7}{18}$ *mi.*

CASE II.

The Minuend a Mixed Number or an Integer.

224. Ex. 1. From $5\frac{3}{4}$ subtract $2\frac{1}{3}$.

Explanation.—We write the subtrahend under the minuend, and reduce the fractional parts to similar fractions. We then subtract the fractional part of the subtrahend from that of the minuend, and the integer of the subtrahend from the integer of the minuend. The result, $3\frac{5}{12}$, is the required difference.

SOLUTION.

$$5\tfrac{3}{4} = 5\tfrac{9}{12}$$
$$2\tfrac{1}{3} = 2\tfrac{4}{12}$$
$$3\tfrac{5}{12}$$

Ex. 2. From $7\frac{3}{8}$ subtract $4\frac{4}{5}$.

EXPLANATION.—After reducing the fractional parts of the given numbers to similar fractions, we find that the fraction of the subtrahend is greater than that of the minuend. We therefore take 1 of the 7, and unite its value ($\frac{40}{40}$) with the $\frac{15}{40}$, thus changing the minuend to $6\frac{55}{40}$. From this we subtract $4\frac{32}{40}$, in the manner shown in Ex. 1.

SOLUTION.

$$7\tfrac{3}{8} = 7\tfrac{15}{40} = 6\tfrac{55}{40}$$
$$4\tfrac{4}{5} = 4\tfrac{32}{40} = 4\tfrac{32}{40}$$
$$2\tfrac{23}{40}$$

Ex. 3. From 15 subtract $4\frac{6}{7}$.

EXPLANATION.—Before subtracting, we reduce 1 of the 15 to sevenths, thus changing the minuend to $14\frac{7}{7}$.

SOLUTION.

$$15 = 14\tfrac{7}{7}$$
$$4\tfrac{6}{7} = 4\tfrac{6}{7}$$
$$10\tfrac{1}{7}$$

PROBLEMS.

17. From $7\frac{5}{9}$ subtract $3\frac{1}{4}$. *Ans.* $4\frac{11}{36}$.

18. From $7\frac{1}{4}$ subtract $3\frac{5}{9}$. *Ans.* $3\frac{25}{36}$.

19. From $8\frac{4}{5}$ subtract $1\frac{1}{12}$. *Ans.* $7\frac{33}{60}$.

20. A merchant bought a cheese which weighed $80\frac{3}{4}$ lb., and sold $23\frac{3}{8}$ lb. of it. How much cheese had he left? *Ans.* $57\frac{3}{8}$ *lb.*

21. A piece of cloth that measured $42\frac{1}{8}$ yd. before it was dressed, shrank $2\frac{3}{5}$ yd in fulling. How many yards did it then contain? *Ans.* $39\frac{24}{40}$.

22. From $19\frac{8}{11}$ subtract $10\frac{2}{3}$.

23. From $5\frac{4}{9}$ subtract $3\frac{1}{2}$.

24. From 7 subtract $3\frac{9}{16}$. *Ans.* $3\frac{7}{16}$.

25. A grocer bought $123\frac{1}{4}$ lb. of butter. After selling $52\frac{5}{8}$ lb. of it, how much had he left? *Ans.* $70\frac{5}{8}$ *lb.*

26. A farmer cut $24\frac{2}{5}$ T. of hay from two meadows, cutting $9\frac{17}{20}$ T. from one of them. How much hay did he cut from the other? *Ans.* $14\frac{14}{20}$ *T.*

27. From $19\frac{6}{7}$ subtract $2\frac{9}{35}$. *Ans.* $18\frac{187}{140}$.

28. If I have $5\frac{1}{4}$ acres of land, and I sell $\frac{4}{5}$ of an acre, how much land have I left? *Ans.* $4\frac{9}{20}$ *acres.*

29. From a bin which contained $4\frac{4}{5}$ bu. of potatoes, a housekeeper used $2\frac{1}{2}$ bu. How many potatoes were left in the bin?

30. A merchant buys boots at $5\frac{1}{3}$ a pair, and sells them at $7 a pair. What are his profits on each pair?

31. A bin that will hold 190 bu., contains $104\frac{3}{4}$ bu. of wheat. How many bushels more will the bin hold? *$85\frac{1}{4}$.*

32. From $17\frac{1}{2}$ subtract $\frac{7}{8}$.

33. From 103 subtract $40\frac{7}{16}$. *$62\frac{9}{16}$.*

34. Wishing to pay my butcher $8\frac{7}{8}$, I hand him a 10-dollar bill. How much change ought I to receive? *$1\frac{1}{8}$.*

SECTION V.

MULTIPLICATION.

CASE I.

The Multiplicand a Fraction.

225. Ex. Multiply $\frac{2}{9}$ by 4.

SOLUTION. $\frac{2}{9} \times 4 = \frac{8}{9}$

EXPLANATION.—Since a fraction is multiplied by multiplying its numerator (See **208**, I), we multiply 2, the numerator of the given fraction, by 4, and under the product write the denominator.

PROBLEMS.

1. Multiply $\frac{3}{22}$ by 5. *$\frac{15}{22}$.*

2. What is the product of $\frac{5}{38}$ multiplied by 7?

3. How much is 3 times $\frac{2}{9}$? *$\frac{6}{9}$, or $\frac{2}{3}$.*

4. If a man and team can plow $\frac{3}{16}$ of an acre in an hour, how much land can they plow in 4 hours? *$\frac{3}{4}$ A.*

5. How much is 7 times $\frac{3}{8}$? *$\frac{21}{8}$, or $2\frac{5}{8}$.*

6. How much will 8 bushels of oats cost, at $\$\frac{2}{5}$ a bushel?

7. How much will 5 cloth caps cost, at $\$\frac{7}{8}$ a piece?

8. How far will a locomotive run in 24 minutes, at the rate of $\frac{5}{8}$ of a mile a minute? *15 mi.*

9. If $\frac{2}{10}$ of an acre will pasture 1 cow through the summer, how many acres will pasture 18 cows? *$16\frac{4}{5}$ A.*

10. A carpenter built 15 lengths of board fence, and each length was $3\frac{2}{3}$ of a rod long. How long was the fence?

11. Multiply $\frac{5}{7}$ by 11, $\frac{17}{20}$ by 9, and $\frac{3}{19}$ by 15.

12. How much is 6 times $\frac{3}{16}$ lb.? 8 times $\frac{7}{12}$ doz.?

$4\frac{7}{8}$ lb.; $4\frac{2}{3}$ doz.

CASE II.

The Multiplier a Fraction.

226. Ex. 1. Multiply 15 by $\frac{2}{3}$; that is, find $\frac{2}{3}$ of 15.

EXPLANATION. — $\frac{2}{3}$ is equal to 2 times $\frac{1}{3}$, and $\frac{1}{3}$ is the result of dividing 1 by 3. Hence, to get $\frac{2}{3}$ of 15, we first divide it by 3 to find $\frac{1}{3}$ of it, and then multiply the result, 5, by 2, to find $\frac{2}{3}$ of it.

SOLUTION.

$15 \div 3 = 5$

$5 \times 2 = 10$

Hence, $15 \times \frac{2}{3} = 10$.

Ex. 2. Multiply 15 by $\frac{5}{12}$; or, find $\frac{5}{12}$ of 15.

EXPLANATION.—We first divide 15 by 12 to find $\frac{1}{12}$ of 15, and then multiply the result, $\frac{15}{12}$, by 5, to find $\frac{5}{12}$ of 15.

SOLUTION.

$15 \div 12 = \frac{15}{12}$

$\frac{15}{12} \times 5 = \frac{75}{12} = 6\frac{3}{12} = 6\frac{1}{4}$

Hence, $15 \times \frac{5}{12} = 6\frac{1}{4}$.

227. Hence, to multiply any number by a fraction, we *Divide the multiplicand by the denominator, and multiply the result by the numerator.*

PROBLEMS.

13. Multiply 18 by $\frac{5}{6}$; that is, find $\frac{5}{6}$ of 18. *15.*

14. What is the product of 45 multiplied by $\frac{5}{9}$?

15. How much is $\frac{4}{7}$ of 42 yards of ribbon? *24 yd.*

16. Multiply 7 by $\frac{6}{11}$. *$4\frac{2}{11}$, or $3\frac{9}{11}$.*

17. I bought 300 lb. of nails, and used $\frac{7}{8}$ of them in building a barn. How many nails did I use? *$262\frac{1}{2}$ lb.*

18. A fat ox weighed 1,172 lb., and, when killed, the beef weighed $\frac{11}{16}$ as much. How much did the beef weigh?

19. Last year I gathered 13 bushels of plums from my garden, and $\frac{3}{4}$ of them were damsons. How many damson plums had I?

20. Multiply 57 by $\frac{7}{15}$, and 23 by $\frac{19}{20}$. *$26\frac{3}{5}$, $21\frac{17}{20}$.*

21. A and B bought a mowing machine for \$145, A paying $\frac{3}{10}$ of the cost, and B $\frac{7}{10}$. How much did each man pay?

A, \$43½; B, \$101½.

CASE III.

Both Factors Fractions.

228. Ex. Multiply $\frac{4}{5}$ by $\frac{2}{3}$; or, find $\frac{2}{3}$ of $\frac{4}{5}$.

EXPLANATION.—We first divide $\frac{4}{5}$ by 3 to find $\frac{1}{3}$ of $\frac{4}{5}$. This we do by multiplying the denominator by 3. (See **208**, II.) We then multiply the result, $\frac{4}{15}$, by 2, to find $\frac{2}{3}$ of $\frac{4}{5}$.

SOLUTION.

$$\frac{4}{5}\times 3 = \frac{4}{15}$$

$$\frac{4}{15}\times 2 = \frac{8}{15}$$

Hence, $\frac{4}{5} \times \frac{2}{3}$, *or* $\frac{2}{3}$ *of* $\frac{4}{5} = \frac{8}{15}$.

229. Hence, to multiply a fraction by a fraction, we

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

The word *of* between fractions signifies multiplication; thus, $\frac{2}{3}$ of $\frac{7}{8} = \frac{2}{3} \times \frac{7}{8}$.

PROBLEMS.

22. Multiply $\frac{5}{8}$ by $\frac{3}{4}$. *$\frac{15}{32}$.*

23. What is the product of $\frac{4}{9} \times \frac{3}{5}$? *$\frac{4}{15}$.*

24. Multiply $\frac{3}{7}$ by $\frac{2}{3}$; $\frac{9}{11}$ by $\frac{4}{7}$.

25. How much will $\frac{3}{4}$ gal. of syrup cost, at \$$\frac{4}{5}$ a gal.? *\$$\frac{3}{5}$.*

26. How much will $\frac{7}{8}$ yd. of brown linen cost, at \$$\frac{4}{5}$ a yd.?

27. How much is $\frac{4}{5}$ of $\frac{11}{16}$ lb.? *$\frac{11}{20}$ lb.*

28. How much is $\frac{3}{4}$ of $\frac{1}{2}$ of an apple? *$\frac{3}{8}$ of an apple.*

29. How much wood is $\frac{3}{4}$ of $\frac{3}{4}$ of a cord?

30. What part of a melon is $\frac{2}{3}$ of $\frac{4}{5}$ of a melon?

31. Three men own a factory in equal shares. How much of the factory does each man own? If one man sells $\frac{1}{2}$ of his share, what part of the factory does he sell? *$\frac{1}{6}$.*

32. A man who owned $\frac{1}{4}$ of a ship sold $\frac{2}{5}$ of his share. How much of the ship did he sell? $\frac{1}{10}$.

33. $\frac{2}{5}$ of $\frac{7}{10}$ = what fraction?

34. What is the product of $\frac{5}{6} \times \frac{2}{3} \times \frac{4}{7}$?

We multiply all the numerators together for the numerator of the product, and all the denominators together for the denominator of the product.

35. What is the product of $\frac{2}{9} \times \frac{5}{8} \times \frac{3}{10}$? $\frac{30}{720}$, or $\frac{1}{24}$.

36. $\frac{1}{2} \times \frac{3}{4} \times \frac{5}{7} \times \frac{4}{5}$ = how many? $\frac{3}{14}$.

37. $\frac{1}{3}$ of $\frac{8}{9}$ is what part of 1? $\frac{8}{27}$.

38. $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{1}{2}$ is what part of 1? $\frac{5}{32}$.

39. What is the product of $\frac{1}{8}$, $\frac{1}{5}$, and $\frac{1}{10}$?

CASE IV.

One or both Factors Mixed Numbers.

230. Ex. Multiply $3\frac{1}{2}$ by $2\frac{2}{3}$.

EXPLANATION.—We first reduce the mixed numbers to improper fractions, and then multiply, as in Case III.

SOLUTION.

$3\frac{1}{2} = \frac{7}{2}$, and $2\frac{2}{3} = \frac{8}{3}$

$\frac{7}{2} \times \frac{8}{3} = \frac{56}{6} = 9\frac{2}{6} = 9\frac{1}{3}$

Hence, $3\frac{1}{2} \times 2\frac{2}{3} = 9\frac{1}{3}$.

PROBLEMS.

40. What is the product of 6 times $3\frac{4}{5}$? $22\frac{4}{5}$.

41. How much will 9 barrels of flour cost, at $\$10\frac{3}{4}$ a barrel?

42. If a man builds $4\frac{2}{3}$ rd. of stone fence in 1 day, how much can he build in 13 days? $60\frac{2}{3}$ *rd.*

43. Multiply 8 by $4\frac{2}{3}$. $37\frac{1}{3}$.

44. How much will $3\frac{3}{4}$ lb. of opium cost, at \$9 a lb.?

45. How much will $\frac{3}{4}$ yd. of vesting cost, at $\$2\frac{1}{8}$ a yd.?

46. Multiply $3\frac{7}{10}$ by $2\frac{3}{8}$. $8\frac{63}{80}$.

47. What is the product of $4\frac{3}{9} \times 3\frac{6}{11}$? $17\frac{4}{9}$.

48. Multiply $9\frac{3}{8}$ by $\frac{7}{12}$; $4\frac{1}{5}$ by $11\frac{2}{3}$. $5\frac{45}{32}$; 49.

49. How many sq. rd. in a field $36\frac{1}{4}$ rd. long, and $21\frac{5}{8}$ rd. wide? $791\frac{41}{64}$.

SECTION VI.

DIVISION.

CASE I.

The Divisor an Integer.

231. Ex. Divide $\frac{8}{9}$ by 4.

EXPLANATION. — To divide $\frac{8}{9}$ by 4, is to find $\frac{1}{4}$ of $\frac{8}{9}$. To do this, we write $\frac{1}{4}$ of $\frac{8}{9}$, and multiply, as in Case III., Multiplication. (See **230**.)

SOLUTION.

$\frac{8}{9} \div 4 = \frac{1}{4}$ of $\frac{8}{9} = \frac{8}{36} = \frac{2}{9}$

Hence, $\frac{8}{9} \div 4 = \frac{2}{9}$.

232. Hence, when the divisor is an integer, we

Write it as the denominator of a fraction with 1 for a numerator, and multiply the given fraction by the fraction thus formed.

PROBLEMS.

1. Divide $\frac{6}{7}$ by 4. $\frac{6}{28}$, *or* $\frac{3}{14}$.
2. Divide $\frac{9}{11}$ by 5. Divide $\frac{24}{25}$ by 16. $\frac{9}{55}$. $\frac{3}{50}$.
3. If 4 lb. of sugar cost $\$\frac{16}{20}$, how much does 1 lb. cost?
4. Six boys gathered $\frac{18}{25}$ of a bushel of chestnuts, and shared them equally. How many chestnuts in 1 boy's share?
5. A butcher packed $\frac{4}{5}$ of a ton of pork in 8 barrels. How much did he put in each barrel? $\frac{1}{10}$ *T.*
6. A seamstress used $\frac{5}{9}$ of a yard of linen in making 9 collars. How much linen did she use for each collar?
7. If 4 oz. of indigo cost $\$\frac{3}{5}$, what is the price of 1 oz.? $\$\frac{3}{20}$.
8. Divide $2\frac{4}{5}$ by 8.

Before dividing, reduce the mixed number to an improper fraction.

9. What is the quotient of $3\frac{3}{4}$ divided by 4? $\frac{15}{16}$.
10. If a teamster draws $3\frac{2}{3}$ cords of stone at 15 loads, how much does he draw at each load? $\frac{11}{45}$ *cd.*

11. What is the quotient of $15\frac{1}{8}$ divided by 18? $\frac{121}{144}$.

12. If 12 boxes of strawberries cost $\$3\frac{1}{5}$, how much does 1 box cost?

13. If 7 men can bind $22\frac{13}{15}$ acres of wheat in one day, how much can 1 man bind? $3\frac{4}{15}$ A.

14. How many times is 9 contained in $41\frac{5}{7}$? $4\frac{19}{63}$.

15. Divide $400\frac{7}{12}$ by 23. $17\frac{5}{12}$.

CASE II.

The Divisor a Fraction.

233. Ex. 1. Divide 5 by $\frac{2}{3}$.

EXPLANATION. — Since the quotient is not changed by multiplying both dividend and divisor by the same number (See **208**, III), we multiply them both by 3, and thus obtain 15 for a new dividend, and 2 for a new divisor. Then, $15 \div 2 = \frac{15}{2} = 7\frac{1}{2}$, the required quotient.

FIRST SOLUTION.

$5 \times 3 = 15$

$\frac{2}{3} \times 3 = 2$

$15 \div 2 = \frac{15}{2} = 7\frac{1}{2}$

Hence, $5 \div \frac{2}{3} = 7\frac{1}{2}$.

SECOND SOLUTION.

$5 \div \frac{2}{3} = 15 \div 2 = \frac{15}{2} = 7\frac{1}{2}$

Hence, $5 \div \frac{2}{3} = 7\frac{1}{2}$.

Ex. 2. Divide $\frac{3}{4}$ by $\frac{2}{5}$.

EXPLANATION. — We first multiply both dividend and divisor by 5, the denominator of the divisor, (See **148**), and then divide the new dividend, $\frac{15}{4}$, by the new divisor, 2, as in Case I.

(See Manual, page 220.)

FIRST SOLUTION.

$\frac{3}{4} \times 5 = \frac{15}{4}$

$\frac{2}{5} \times 5 = 2$

$\frac{15}{4} \div 2 = \frac{1}{2}$ of $\frac{15}{4} = \frac{15}{8} = 1\frac{7}{8}$

Hence, $\frac{3}{4} \div \frac{2}{5} = 1\frac{7}{8}$.

SECOND SOLUTION.

$\frac{3}{4} \div \frac{2}{5} = \frac{15}{4} \div 2 = \frac{1}{2}$ of $\frac{15}{4} = \frac{15}{8} = 1\frac{7}{8}$

Hence, $\frac{3}{4} \div \frac{2}{5} = 1\frac{7}{8}$.

234. From these examples it will be seen that, to divide by a fraction, we

Multiply the dividend by the denominator of the divisor, and divide the result by the numerator.

PROBLEMS

16. Divide 3 by $\frac{2}{5}$. $7\frac{1}{2}$.

17. Divide 7 by $\frac{1}{3}$, and 6 by $\frac{3}{4}$. *21; 8.*

18. At $\$\frac{4}{5}$ a bushel, how many bushels of apples can be bought for \$5? $6\frac{1}{4}$ *bu.*

19. The dividend is 9, and the divisor is $\frac{2}{3}$. What is the quotient? $40\frac{1}{2}$.

20. What is the quotient of $\frac{7}{10}$ divided by $\frac{3}{8}$? $1\frac{13}{15}$.

21. How many times is $\frac{2}{7}$ contained in $\frac{15}{16}$?

22. How much ribbon, at $\$\frac{3}{10}$ a yard, can be bought for $\$\frac{3}{4}$?

23. If a horse walks a mile in $\frac{4}{15}$ of an hour, how far will he walk in 8 hours? *30 miles.*

24. At $\$\frac{2}{3}$ a pound, how much candy can be bought for $\$\frac{5}{8}$?

25. Divide $3\frac{1}{5}$ by $4\frac{2}{3}$.

Before dividing, reduce the mixed numbers to improper fractions. Thus, $3\frac{1}{5} \div 4\frac{2}{3} = \frac{16}{5} \div \frac{14}{3} = \frac{48}{70} = \frac{24}{35}$.

26. What is the quotient of $8 \div 2\frac{4}{5}$? $2\frac{6}{7}$.

27. Divide $\frac{17}{24}$ by $3\frac{2}{5}$. $\frac{5}{24}$.

28. The dividend is $5\frac{4}{7}$, and the divisor is $\frac{3}{8}$. What is the quotient? $14\frac{6}{7}$.

29. A shoe-dealer paid \$150 for a case of boots, at $\$6\frac{1}{4}$ a pair. How many pairs of boots were in the case?

30. A man whose daily wages were $\$2\frac{1}{4}$, received at the end of the week $\$12\frac{3}{5}$. How many days had he worked? $5\frac{3}{5}$.

31. I paid $\$\frac{33}{40}$ for $5\frac{1}{2}$ lb. of rice. What was the price per lb.?

32. How many rolls of wall-paper, each containing $4\frac{1}{2}$ sq. yd., will be required to cover $58\frac{1}{2}$ sq. yd. of wall? *13.*

33. What is the quotient of $\frac{2}{7}$ divided by $\frac{5}{9}$?

34. Divide $\frac{7}{8}$ by $\frac{19}{20}$; $\frac{19}{20}$ by $\frac{7}{8}$. $\frac{35}{38}$; $1\frac{3}{35}$.

35. What is the quotient of $15\frac{2}{7} \div 24\frac{1}{2}$? $\frac{214}{343}$.

SECTION VII.

CANCELLATION.

CASE I.

In Multiplication.

235. What is the product of $\frac{5}{8}$, $\frac{4}{7}$, and $\frac{21}{22}$?

FIRST SOLUTION.

$$\frac{5}{8} \times \frac{4}{7} \times \frac{21}{22} = \frac{420}{1232} = \frac{105}{308} = \frac{15}{44}$$

SECOND SOLUTION.

$$\frac{5}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{7} \times \frac{\cancel{21}^{3}}{22} = \frac{15}{44}$$

EXPLANATION.—In the First Solution we reduce the product, $\frac{420}{1232}$, to its lowest terms, by dividing both dividend and divisor by 4, and both terms of the result thus obtained by 7. But, in the Second Solution, we divide the numerator, 4, and the denominator, 8, by 4, and the numerator, 21, and the denominator, 7, by 7. We then multiply the remaining numbers in the numerators together for the numerator of the product, and the remaining numbers in the denominators together for the denominator of the product. The results in the two solutions are the same. (See Manual, page 220.)

236. The process of dividing a numerator and a denominator by any number, either in the same fraction, or in fractions which are to be multiplied together, is *Cancellation*.

In multiplication of fractions, whenever a numerator and a denominator contain the same factor, the process can be shortened by Cancellation. In this case, the product will always be in its lowest terms.

PROBLEMS.

1. Multiply $\frac{4}{7}$ by $\frac{3}{8}$; $\frac{5}{6}$ by $\frac{9}{10}$; and $\frac{5}{8}$ by $\frac{14}{15}$. $\frac{3}{14}$, $\frac{3}{4}$, *and* $\frac{7}{12}$.
2. What is the product of $\frac{4}{5}$, $\frac{3}{8}$, and $\frac{5}{6}$? $\frac{1}{6}$.
3. Multiply $\frac{1}{2}$, $\frac{4}{9}$, $\frac{3}{5}$, and $\frac{6}{7}$ together.

4. How much is $\frac{1}{8}$ of $\frac{6}{7}$ of $\frac{4}{9}$ of $\frac{7}{10}$?

5. $\frac{5}{6} \times 2\frac{2}{3} \times 1\frac{1}{5} \times \frac{3}{8}$ = what fraction? $\frac{2}{3}$.

6. $\frac{7}{10}$ of $\frac{3}{9}$ of $\frac{5}{6}$ of $\frac{3}{4}$ is what part of 1? $\frac{7}{48}$.

7. How much must I pay for $6\frac{3}{12}$ ft. of gas-pipe, at $\$\frac{3}{8}$ a ft.?

8. How many sq. ft. of oil-cloth will be required to cover an office table $3\frac{3}{4}$ ft. long, and $2\frac{2}{3}$ ft. wide? *10.*

9. How much will $1\frac{9}{16}$ yd. of linen cost, at $\$\frac{1}{2}\frac{6}{5}$ a yd.?

10. How much must I pay for cutting $\frac{2}{3}$ of $\frac{1}{4}$ cd. of wood, at $\frac{4}{5}$ of $\$\frac{15}{16}$ a cord? $\$\frac{1}{8}$.

CASE II.

In Division.

237. Ex. 1. Divide $\frac{5}{9}$ by $\frac{3}{4}$.

EXPLANATION. — In this Solution we have multiplied the numerator of the dividend by the denominator of the divisor ($5 \times 4 = 20$), and the denominator of the dividend by the numerator of the divisor ($9 \times 3 = 27$).

FIRST SOLUTION.

$$\frac{5}{9} \div \frac{3}{4} = \frac{20}{9} \div 3 = \frac{1}{3} \text{ of } \frac{20}{9} = \frac{20}{27}$$

If we change the places of the terms of the divisor, and multiply the dividend by $\frac{4}{3}$, the fraction thus formed, we shall multiply the same numbers together as in the First Solution. This is shown in the Second Solution. Hence,

SECOND SOLUTION.

$$\frac{5}{9} \div \frac{3}{4} = \frac{5}{9} \times \frac{4}{3} = \frac{20}{27}$$

238. In dividing by a fraction, we may

Change the places of the terms of the divisor, and multiply the dividend by the fraction thus formed.

Ex. 2. Divide $\frac{5}{12}$ by $\frac{15}{16}$.

EXPLANATION.—We first invert the divisor — that is, change the places of the terms —and then proceed as in Case I.

SOLUTION.

$$\frac{5}{12} \div \frac{15}{16} = \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{15}}} = \frac{4}{9}$$

PROBLEMS.

11. Divide $\frac{3}{4}$ by $\frac{5}{6}$, and $\frac{5}{8}$ by $\frac{3}{4}$. *$\frac{9}{10}$, and $\frac{5}{6}$.*

12. How many times is $\frac{9}{10}$ contained in $2\frac{3}{20}$? *$2\frac{7}{18}$.*

13. I paid \$58$\frac{1}{8}$ for building $3\frac{3}{4}$ rods of door-yard fence. How much did it cost me a rod? *\$15$\frac{1}{2}$.*

14. Divide $1\frac{7}{8}$ by $4\frac{1}{6}$?

15. How many brooms can be made from $22\frac{1}{2}$ lb. of broom-corn, if $\frac{5}{16}$ lb. is used for 1 broom? *6 doz.*

16. A hardware dealer paid \$45 for sheep shears, at \$$\frac{9}{10}$ a pair. How many pairs did he buy?

17. How many boxes, each $5\frac{1}{2}$ in. long, $4\frac{1}{4}$ in. wide, and 3 in. deep, will be required to hold as much as one box $8\frac{1}{2}$ in. long, $8\frac{1}{4}$ in. wide, and 6 in. deep? *6.*

18. Divide $\frac{3}{11}$ of $\frac{2}{9}$ by $\frac{1}{2}$ of $\frac{4}{9}$ of $\frac{3}{8}$. *$\frac{8}{11}$.*

SECTION VIII.

PROBLEMS IN FRACTIONS.

1. A provision dealer sold 7 barrels of pork for \$$113\frac{3}{4}$. What was the price per barrel? *\$16$\frac{1}{4}$.*

2. How many cubic feet are there in a block of marble $4\frac{1}{2}$ ft. long, $1\frac{3}{4}$ ft. wide, and $\frac{1}{2}$ ft. thick? *$3\frac{15}{16}$.*

3. How much will 1 pound of sugar cost, if $4\frac{1}{2}$ pounds cost \$$\frac{27}{40}$? *\$$\frac{3}{20}$.*

4. An errand boy earned \$$\frac{4}{5}$ on Monday, \$$\frac{5}{8}$ on Tuesday, and \$$\frac{7}{8}$ on Wednesday. How much did he earn in the three days?

5. Reduce $\frac{48}{84}$ to its lowest terms.

6. In $17\frac{4}{11}$, how many elevenths?

7. How much can a man earn in $\frac{3}{4}$ of a day, at \$$1\frac{5}{8}$ a day?

8. A grocer buys cheese at \$$\frac{19}{100}$ a pound, and sells it at \$$\frac{1}{4}$ a pound. How much does he gain per pound?

9. If you pronounce 63 words each minute in reading aloud, how long will it take you to read a chapter containing 5,796 words? *1 h. 32 min.*

10. The sum of two numbers is $83\frac{5}{9}$, and one of them is $47\frac{1}{6}$. What is the other? *$36\frac{7}{18}$.*

11. The difference of two numbers is $2\frac{4}{5}$, and the greater number is $61\frac{7}{10}$. What is the less number? *$58\frac{9}{10}$.*

12. The difference of two numbers is $\frac{3}{16}$, and the less number is $\frac{5}{24}$. What is the greater number? *$\frac{61}{432}$.*

13. How many lights of glass, each containing $\frac{5}{6}$ of a square foot, are there in a box of glass which contains 50 square feet?

14. Reduce $3\frac{5}{11}$ to forty-fourths.

15. Reduce $\frac{4}{7}$, $\frac{4}{5}$, and $\frac{4}{9}$ to similar fractions.

16. Reduce $\frac{132}{477}$ to a mixed number?

17. If $11\frac{2}{3}$ doz. eggs cost $\$3\frac{4}{15}$, how much will $13\frac{3}{4}$ doz. cost?

18. How many pieces of stone flagging, each $2\frac{3}{4}$ ft. square, will it take to make a sidewalk $148\frac{1}{2}$ ft. long and $5\frac{1}{2}$ ft. wide?

19. What part of $19\frac{3}{8}$ is $18\frac{1}{12}$? *$\frac{14}{15}$.*

20. One Saturday morning, Frank caught five fish, the first of which weighed $3\frac{7}{16}$ lb., the second $3\frac{1}{4}$ lb., the third $2\frac{1}{4}$ lb., and the fourth and fifth each $2\frac{5}{8}$ lb. What was the weight of all?

21. A farmer sold $\frac{2}{5}$ of his land, and afterward bought $37\frac{3}{4}$ acres. He then had $112\frac{3}{20}$ acres. How much land had he at first? *124 A.*

22. What is the difference between $\frac{7}{12}$ of $18\frac{3}{4}$ and $\frac{2}{3}$ of $17\frac{3}{4}$?

23. How much will a turkey weighing $8\frac{11}{16}$ pounds cost, at $\$\frac{6}{25}$ a pound? *$\$2\frac{17}{200}$.*

24. If you sleep $7\frac{5}{12}$ hours each day, how much time do you spend in sleep in 1 year? *16 wk. $19\frac{1}{2}$ h.*

25. A grocer bought a barrel of sugar, and after selling $\frac{3}{16}$ of it, $\frac{1}{6}$ of it, and $\frac{5}{24}$ of it, he had 119 pounds left. How many pounds were in the barrel at first? *272.*

26. Reduce $\frac{91}{119}$ to its lowest terms.

27. A laborer laid $189\frac{1}{16}$ rods of tile-drain in $13\frac{3}{4}$ days. How much did he lay each day? *$13\frac{3}{4}$ rods.*

28. I paid 12 cents a pound for $11\frac{3}{8}$ pounds of beef, but $\frac{2}{7}$ of it was bone. As the bone was worth nothing, how much a pound did the meat cost me? *$16\frac{4}{5}$ cents.*

29. If a man can earn $17.50 in 26 days, when he works 10 hours a day, how much can he earn in 19 days, when he works 12 hours a day? *$62.70.*

30. How much will 375 pounds of bone-dust cost, at $38 a ton? *$7.125.*

31. William is $\frac{3}{10}$ as old as his father, his father is $\frac{5}{9}$ as old as his grandfather, and his grandfather is 72 years old. How old is William? *12 years.*

32. Two men dug a well in five days, digging $11\frac{1}{2}$ ft. the first day, $6\frac{3}{4}$ ft. the second, $5\frac{1}{3}$ ft. the third, $3\frac{3}{4}$ ft. the fourth, and $3\frac{5}{12}$ ft. the fifth. How deep was the well?

33. If Andrew can run $968\frac{3}{4}$ rods in 25 minutes, and Richard can run 963 rods in 24 minutes, which can run the faster?

34. If they start together, and run in the same direction for $18\frac{1}{2}$ minutes, how far will they be apart? *$25\frac{7}{10}$ rods.*

35. The minuend is $201\frac{4}{7}$, and the subtrahend $\frac{19}{20}$. What is the remainder?

36. The dividend is $\frac{7}{12}$ of $1\frac{5}{7}$, and the divisor is $\frac{5}{9}$ of $87\frac{3}{4}$. What is the quotient? *$\frac{4}{7}\frac{9}{3}\frac{9}{5}$.*

37. The sum of three numbers is $357\frac{5}{16}$, and two of them are $265\frac{5}{8}$ and $\frac{11}{12}$. What is the third number? *$90\frac{37}{48}$.*

38. A sewing girl earns $\$\frac{5}{8}$ a day, and pays $\$2\frac{1}{2}$ a week for her board. How much money will she have at the end of 13 weeks, after paying for her board? *$\$16\frac{1}{4}$.*

39. How many square feet are there in a board 16 ft. long and $\frac{5}{6}$ ft. wide?

40. Four boys spent one Saturday in gathering walnuts. On dividing them, Albert received $\frac{1}{3}$ of all they gathered, Robert $\frac{4}{15}$, Thomas $\frac{3}{10}$, and David $17\frac{1}{2}$ quarts. How many walnuts did each of the other boys have?

41. How many walnuts did they gather in all?

2 bu. 1 pk. 3 qt.

42. If $12\frac{4}{5}$ acres of wheat can be cut with a reaper in 1 day, how many acres can be cut in $3\frac{5}{12}$ days? *$43\frac{11}{12}$.*

CHAPTER V.

PERCENTAGE.

SECTION I.

NOTATION AND NUMERATION.

239. In business transactions, hundredths of any thing or number are commonly called *Per Cent.* Thus, .04 is 4 per cent., .09 is 9 per cent., .48 is 48 per cent. Hence,

I. *Any per cent. of a thing or number is so many hundredths of that thing or number; and,*

II. *Per cent. may always be written as a decimal.*

240. This character, %, placed after a number, is the *Commercial Sign* of per cent. Thus, 15% signifies 15 hundredths, and is read 15 per cent.

EXERCISES.

1. Write, decimally, 8 per cent., 3 per cent., 5 per cent.

2. Write 14 per cent., 17 per cent., 28 per cent., 33 per cent., 40 per cent., 65 per cent., 10 per cent.

3. Read the following decimals as per cent.: .07, .19, .30, .42, .50, .69, .06, .99, .75.

4. First read, and then write, decimally, 12%, 29%, 63%, 90%.

5. Read, and then write, 1%, 9%, 10%, 56%, 47%.

6. Write, with the sign of per cent., the decimals in Exercise 3.

241. 125% is written, decimally, 1.25, 308% is written 3.08.

VALUES OF FRACTIONS IN DECIMALS.

$\frac{1}{2} = .5$, and $\frac{1}{2}\% = .005$	$\frac{1}{8} = .125$, and $\frac{1}{8}\% = .00125$
$\frac{1}{4} = .25$, and $\frac{1}{4}\% = .0025$	$\frac{3}{8} = .375$, and $\frac{3}{8}\% = .00375$
$\frac{3}{4} = .75$, and $\frac{3}{4}\% = .0075$	$\frac{5}{8} = .625$, and $\frac{5}{8}\% = .00625$
$\frac{1}{5} = .2$, and $\frac{1}{5}\% = .002$	$\frac{7}{8} = .875$, and $\frac{7}{8}\% = .00875$

$12\frac{1}{2}\%$ is written $.12\frac{1}{2}$ or .125, $\frac{1}{4}\%$ or .25% is written $.00\frac{1}{4}$ or .0025. Hence,

I. *To express per cent. decimally, two decimal figures are always required.*

II. *To express more than 99 per cent., an integer or a mixed decimal number is required.*

III. *To express parts of 1 per cent., decimal figures or fractions at the right of hundredths are required.*

EXERCISES.

7. Write, decimally, 114%, 159%, 237%, 475%, 108%, 200%.
8. Read .038, .0425, .165, $.43\frac{3}{8}$, $.31\frac{1}{4}$, $.05\frac{1}{2}$.
9. Write $6\frac{1}{4}\%$, $16\frac{7}{8}\%$, $10\frac{1}{5}\%$, 18.7%, 22.5%, 31.25%, $\frac{1}{4}\%$, $\frac{7}{8}\%$.

SECTION II.

GENERAL APPLICATIONS.

242. Per cent. may be applied to any number, great or small, concrete or abstract.

243. The process of finding any per cent. of a number is *Percentage.* The result of the computation is also called *Percentage.*

244. The number expressing the per cent. is the *Rate,* or *Rate Per Cent.*

245. The number on which the percentage is computed is the *Base.*

Ex. How much is 15% of 125?

EXPLANATION.—Since 15% of a number is .15 of that number, we multiply 125 by .15, as in Multiplication of Decimals.

SOLUTION.

125 Base.
.15 Rate.
625
125
18.75 Percentage.

246. *Rule for Percentage.*

Multiply the base by the rate expressed decimally.

PROBLEMS.

1. Of a flock of 125 sheep, 4% were killed by dogs. How many sheep were killed? *5.*

2. At a school of 125 pupils, the average daily attendance is 88% of the whole number. What is the daily attendance?

3. From a cask of 44 gallons of oil, 5% leaked out. How much oil leaked out? *2.2 gallons.*

4. Crystal Lake covers 87% of a square mile. How many acres does it cover? *556.8.*

5. Of a cargo of 14,865 bushels of wheat, 12% was injured by water. How many bushels were injured? *1783.8.*

6. A piece of cotton cloth containing 42 yards, shrank 6% in bleaching. How many yards did it then contain?

100% of a number is the whole of it. $100\% - 6\% = 94\%$; and 94% of 42 yd. $= 39.48$ *yd.*

7. A fruit-grower set out 250 peach-trees, but 18% per cent. of them died. How many trees lived? *205.*

8. A steamboat company bought 5,280 cords of wood, of which 21% was hickory, 33% was beach, and the balance was maple. How many cords of each kind did they buy?

9. A man husked 284 bushels of corn, and received $12\frac{1}{2}\%$ of it for his labor. How much corn did he receive?

SECTION III.

COMMISSION.

247. A person who buys and sells goods for another, receiving for his services a certain % on the value of the goods bought or sold, is an *Agent or Commission-Merchant.*

248. The sum paid the agent or commission-merchant for his services is *Commission.*

Ex. A real-estate agent sold a house and lot for $2,275. How much did he receive for making the sale, at 2% commission?

SOLUTION.

$2275 Base.
.02
$45.50 Commission.

EXPLANATION.—Since he received a commission of 2%, he received .02 of $2,275, which is $45.50.

PROBLEMS.

1. An auctioneer sold farm produce and stock to the amount of $1,185, and charged 5% commission. How much did he receive? *$59.25.*

2. A commission-merchant sold 500 barrels of flour for $4,750. How much was his commission, at 2%? *$95.*

3. A land agent paid out $2,970 for western land, on a commission of 5%. How much did his commission amount to? *$148.50.*

4. A canvassing agent received 40% for selling pictures. His receipts were $106 in Jan., $95 in Feb., and $126 in Mar. How much did he earn each month?

5. My agent purchases for me 2,680 bu. of wheat, at $1.125 a bushel, on a commission of $2\frac{1}{2}\%$. How much is his commission? *$75.37\frac{1}{2}$.*

6. I sold 45,000 pounds of cotton, at $.17 a pound, on a commission of $\frac{3}{8}\%$. How much did my commission amount to? *$28.68\frac{3}{4}$.*

SECTION IV.

INSURANCE.

249. A company that, for a certain sum, agrees to pay for property, if it is destroyed by fire or lost at sea, is an *Insurance Company;* and

250. The agreement is an *Insurance.*

251. The sum of money paid to secure an insurance is the *Premium.*

252. The premium is usually computed at a certain per cent. on the sum for which the property is insured.

PROBLEMS.

1. How much will it cost to insure a dwelling for \$850, at 2%?

SOLUTION OF PROBLEM 1.

\$850
.02
\$17.00 Premium.

2. What premium must a carpenter pay to get his shop insured for \$975, at 3%? *\$29.25.*

3. A hotel and furniture are insured for \$45,000, at 2%. What is the annual or yearly premium? *\$900.*

4. A cargo of flour, shipped from New York to Liverpool, is insured for \$9,000, at $\frac{1}{2}$%. How much is the premium?

5. How much will it cost a year to insure a school-house for \$1,250, at an annual premium of $\frac{3}{4}$%?

6. How much will it cost for 3 years? *\$28.12½.*

7. An agent insured a flouring-mill for \$6,750, at 1¼%. What premium did he receive? *\$84.37½.*

8. A ship was insured for a voyage from Boston to San Francisco for \$32,500, at 2¼%. What was the premium?

SECTION V.

PROFIT AND LOSS.

253. When anything is sold for more than it cost, the gain or increase in its value is *Profit;* and

254. When it is sold for less than it cost, the decrease in its value is *Loss.*

255. Profit and Loss are commonly estimated at some per cent. upon the cost price of the property.

Ex. A merchant bought broadcloth at \$2.50 a yard, and sold it so as to gain 40%. How much did he gain on a yard? At what price per yard did he sell it?

EXPLANATION.—At 40%, the gain on $2.50 is .40 of $2.50, which is $1.00; and $2.50, the cost price, + $1.00, the gain, = $3.50, the selling price.

Had the cloth been sold at a loss of 40%, the $1.00 would be loss, and the selling price would be found by subtracting the loss from the cost.

SOLUTION.

$2.50 Cost Price.
.40
$1.00 Gain.
2.50
$3.50 Selling Price.

PROBLEMS.

1. A mechanic built a house at a cost of $1,500, and in selling it, gained 12%. How much did he gain? *$180.*

2. A grocer sells sugar that costs him $.12 a pound, at 25% profit. How much does he make on 1 pound? *$.03.*

3. In selling kerosene that costs $.48 a gallon, at 56¼% profit, how much is the gain on 1 gallon? *$.27.*

4. I bought a horse for $180, and sold him at a loss of 12½%. How much did I lose by the bargain? *$22.50.*

5. A farmer paid $6,000 for 75 acres of land, and sold it at 7½% less than he gave for it. How much was his loss?

6. A merchant sells delaines which cost $.31¼ a yard, at a loss of 20%. How much does he lose on a yard? *$.06¼.*

7. At what price must I sell a sewing-machine that cost me $45, to gain 25%? *$56.25.*

8. A carpet dealer bought carpeting at $1.25 a yard, and marked it to be sold at 60% above cost. At what price did he mark it? *$2.00.*

9. If coal that costs $6.75 a ton is sold at 24% below cost, what is the selling price? *$5.13.*

10. What price must I put on a bureau that cost me $15 to manufacture, in order to gain 40%?

11. A man paid $300 for a village lot, $1,600 for building a house upon it, and $200 for other improvements. He then offered to sell the property at 10% below cost? What was his asking price? *$1,890.*

SECTION VI.

INTEREST.

256. If a person hires a house or a farm, he pays something for the use of it.

So, also, if a person borrows money, when he pays the debt, he pays the sum borrowed and an additional sum for the use of it; and

If a person pays a debt after it is due, he pays not only the amount of the debt, but an additional sum for the use of that amount.

257. The sum paid for the use of money is *Interest.*

258. The sum for the use of which the interest is paid is the *Principal.*

259. The sum of the principal and interest is the *Amount.*

260. The rate, or the interest of 1 dollar for 1 year, is always some per cent. of the principal.

In most States, the rate per cent. is established by law.

CASE I.

Interest for Years.

261. Ex. 1. What is the interest of \$395 for 1 year, at 6%?

SOLUTION.

\$395	Principal.
.06	Rate.
\$23.70	Interest.

EXPLANATION.—At 6%, the interest is .06 of the principal, and .06 of \$395 is \$23.70.

Ex. 2. What is the interest of \$135.25 for 3 years, at 6%?

SOLUTION.

\$135.25	
.06	
\$8.1150	Int. for 1 yr.
3	
\$24.345	Int. for 3 yr.

EXPLANATION.—The interest for 3 years is 3 times as much as the interest for 1 year, or 3 times \$8.115, which is \$24.345.

Ex. 3. What is the amount of $350 for 5 years, at 7%?

SOLUTION.

$350
.07
$24.50
5
$122.50 Interest.
350 Principal.
$472.50 Amount.

EXPLANATION. — The interest of $350 for 5 years, at 7%, is $122.50; and $122.50 (the interest), + $350 (the principal), = $472.50, the amount.

PROBLEMS.

1. What is the interest of $25 for 1 year, at 6%? *$1.50.*
2. Find the interest of $132 for 1 year, at 5%. *$6.60.*
3. Find the interest of $76.50 for 1 year, at 7%. *$5.355.*
4. What is the interest of $216.25 for 3 years, at 8%? *$51.90.*
5. A man paid a debt of $188.65, with interest at 10%, 1 year after it was due. What amount did he pay? *$207.515.*
6. What is the amount of $3,750 for 3 years, at 5%?
7. February 11, 1864, I borrowed $250. How much did the debt amount to, February 11, 1867, interest at 6%?
8. What is the interest of $560.10 for 2 years, at $6\frac{1}{2}$%?

CASE II.

Interest for Months.

262. Ex. What is the interest of $84.18 for 7 months, at 6%?

SOLUTION.

$84.18
.06
$5.0508 Int. for 1 yr.
7
$35.3556 | 12
$2.9463 Int. for $\frac{7}{12}$ yr.

EXPLANATION.—The interest for 1 year is $5.0508; and the interest for 7 months, or $\frac{7}{12}$ of a year, is $\frac{7}{12}$ of $5.0508, which is $2.9463.

To find $\frac{7}{12}$ of $5.0508, the interest for 1 year, we multiply it by the numerator, 7, and divide the product by the denominator, 12 (see **227**); but 7 is the given number of months, and 12 is the number of months in a year. Hence,

To find the interest for any number of months,

Multiply the interest for 1 year by the given number of months, and divide the product by 12.

(See Manual, page 220.)

The answers to the remaining problems in this Chapter are given in accordance with the principle stated in Art. **159.**

PROBLEMS.

9. What is the interest of \$152.17 for 3 months, at 6%?

10. What is the interest of \$18.72 for 5 months, at 4%?

11. Find the interest of \$584.24 for 1 yr. 4 mo., at 7%. (1 yr. 4 mo. = 16 mo.) *\$54.53.*

12. If I have \$1876.50 at interest for 2 yr. 5 mo., at 6%, how much interest will I receive? *\$272.09.*

13. What is the amount of \$294.25 for 6 mo., at 5%?

14. If I give my note for \$275, Jan. 11, 1867, and pay it Feb. 11, 1868, with 7% interest, how much do I pay?

15. If I borrow \$732, at 12% interest, and pay the debt in 1 month, how much do I pay? *\$739.32.*

CASE III.

Interest for Days.

263. In computing interest, 30 days are called a month. Hence,

Every 3 days are $\frac{1}{10}$, or .1 of a month, and
Every 1 day is $\frac{1}{3}$ of $\frac{1}{10}$, or .0$\frac{1}{3}$ of a month.

264. Ex. 1. What is the interest of \$675 for 18 days, at 7%?

SOLUTION.

\$675
.07
\$47.25
.6
\$28.350 | 12
\$2.3625

EXPLANATION.—Since every 3 days are .1 of a month, 18 days are .6 of a month. We therefore multiply \$47.25, the interest for 1 year, by .6, and divide the product by 12, as in Case II.

Ex. 2. What is the interest of \$169.44 for 1 yr. 3 mo. 17 da., at 6%?

EXPLANATION.—Since 1 yr. 3 mo. are 15 mo., and 17 da. are .5⅔ of a mo., the whole time is 15.5⅔ mo. We therefore multiply \$10.1664, the interest for 1 year, by 15.5⅔, and divide the product by 12, as in Case II., and the result, \$13.19, is the interest required. In multiplying by ⅔, we take ⅓ of the multiplicand twice.

SOLUTION.

\$169.44
.06
\$10.1664
15.5⅔
33888
33888
508320
508320
101664
\$158.25696 | 12
\$13.188

Years, months, and days can be expressed as months and tenths of a month.

PROBLEMS.

16. What is the interest of \$116.25 for 24 days, at 6%? *\$.465.*

17. If I borrow \$819 for 20 days, at 8%, how much interest must I pay? *\$3.64.*

18. How much interest will I have to pay, at 7%, on a loan of \$1296, for 9 mo. 15 da.? *\$71.82.*

19. What is the interest of \$936 for 3 yr. 2 mo. 29 da., at 10%? *\$303.94.*

20. Find the interest of \$718 for 1 yr. 14 da., at 6%.

21. What is the interest of \$48, from Nov. 23, 1866, to Dec. 8, 1867, at 7%? *\$3.50.*

265. *General Rule for Interest.*

I. For 1 year, *Multiply the principal by the rate.*

II. For 2 or more years, *Multiply the interest for 1 year by the number of years.*

III. For any other time, *Multiply the interest for 1 year by the time expressed in months and tenths of a month, and divide the product by 12.*

IV. For the amount, *Add the interest to the principal.*

PROBLEMS.

27. How much interest must I pay for the use of $756.50 for 5 years, at 7%? *$264.775.*

28. A note of $1824.75, dated Oct. 9, 1867, was paid Oct. 9, 1868, with interest at 6%. What was the amount paid?

29. There is a mortgage on my house and lot for $1244, with interest at 7%. How much interest is due annually?

30. A teacher in St. Louis bought a house and lot for $3750, and paid for it at the end of 3 years, with interest at 6%. What amount did he pay? *$4425.*

31. What is the interest of $752.50 for 8 months, at 9%?

32. What is the interest of $87.36, at 10%, from Feb. 10, 1866, to Oct. 10, 1867? *$14.56.*

33. A young man bought a watch for $85, and paid for it in 9 months, with interest at 7%. How much did he pay?

SECTION VII.

PROBLEMS IN PERCENTAGE.

1. On opening a box of 128 panes of glass, a glazier found $6\frac{1}{4}$% of them broken. How many panes were broken? *8.*

2. I had $1,756 in accounts, and I have collected 78% of them. What sum have I collected? *$1369.68.*

3. A sewing-machine agent in one year sold 285 machines, at $60 each. How much did his commission amount to, at 25%? *$4,275.*

4. A produce dealer sold 1,756 pounds of butter, at $.28 a pound, for a country merchant. How much was his commission, at 5%? *$24.584.*

5. At $1\frac{1}{2}$%, what premium must I pay for an insurance of $1,500 on my dwelling? *$22.50.*

6. At $\frac{3}{4}$%, what premium must a farmer pay annually for an insurance of $800 on his house, $900 on his barns, and $1,100 on his grains and hay?

7. How much premium will he pay in 5 years? *$105.*

8. A country store is insured for $1,250, and the goods are insured for $3,250, at $1\frac{5}{8}\%$. How much is the annual premium on the store and goods? *$73.12½.*

9. A farmer bought a reaper for $140, and gave his note in payment for it. He paid the note 4 years afterward, with interest at 10%. How much did he pay? *$196.*

10. A note for $417.12, dated Aug. 23, 1867, was due Sept. 23, 1868, with interest at 8%. What sum was required to take up the note? *$453.27.*

11. What is the amount of $10843.40 for 1 yr. 7 mo., at 10%? *$12560.27.*

12. Find the interest of $162 for 4 mo. 9 da., at $6\frac{1}{2}\%$. *$3.77.*

13. What is the interest on $3,000 for 1 yr. 5 mo. 2 da., at 7.3%? *$311.47.*

14. What is the amount of $792 for 17 da., at 5%? *$793.87.*

15. Dec. 15, 1864, a farmer mortgaged his farm for $4,850, and Sept. 15, 1867, he paid the mortgage, with 6% interest. What amount did he pay? *$5650.25.*

16. In a school of 140 pupils 45% of them are boys, and 55% are girls. How many boys belong to the school?

17. How many girls are in the school? *77.*

18. A young man invested $3,450 in business, and lost 17% of it the first year. How much money had he left in the business? *$2863.50.*

19. A produce dealer bought wheat at $1.60 per bushel, and sold it at a loss of 10%. At what price did he sell it?

20. If I invest $2,500 in bank stock, and sell it at an advance of 6%, for what sum do I sell?

21. April 11, 1867, a man bought a canal boat for $1,140, and Nov. 11, 1868, he paid for it, with interest at 7%. How much did he pay? *$1266.35.*

CHAPTER VI.

MISCELLANEOUS PROBLEMS;

Embracing all the Principles and Methods of Computation in the Preceding Chapters. (See Manual, page 220.)

1. A drover paid $35 each for 32 head of cattle, and $43 each for 23 head. How many cattle did he buy, and how much did they cost him? *They cost him $2,086.*

2. A farmer sold a farm of 96.23 acres at $85 an acre, and afterward bought another of 123.47 acres for $52.50 an acre. Which farm came to the more money? How much the more? *The first farm; $1697.37½.*

3. A dealer in garden seeds put up 8 bu. 2 pk. 5 qt. of peas in papers holding 1 pt. each. How many papers did he have? *554.*

4. How many years, months, and days old are you to-day?

5. How long a time has elapsed since the Declaration of American Independence, which was made July 4, 1776?

6. How many bushels of lime can be bought for $7.35, at $.20 a bushel? *36.75.*

7. A lady paid $52.50 for 28 yards of carpeting. What was the price per yard?

8. At what price per lb. must I sell 84.5 lb. of raisins, to receive $37.18 for them? *$.44.*

9. If 1 lb. of cheese can be made from $3\frac{3}{4}$ qt. of milk, how many lb. can be made from $35\frac{5}{8}$ qt.? *9½.*

10. A gardener has 12 hives of bees, and last summer he obtained $8\frac{1}{4}$ lb. of honey from each hive. How much honey did he get in all?

11. A hop grower bought 3,875 hop-poles, at $22 a thousand. How much did he pay for them?

Since 1,000 hop-poles cost $22, 1 hop-pole must cost $\frac{1}{1000}$ (or .001) of $22, which is $.022; and 3,875 hop-poles, at $.022 apiece, must cost 3,875 times $.022, which is *$85.25.*

12. A nursery-man sold 144 apple-trees, at $12 a hundred. How much did he receive for them? *$17.28.*

13. In building a house, I used 21,375 feet of clapboards, for which I paid $35 per thousand. How much did they cost me? *$748.12½.*

14. What will be the cost of 7 pieces of sheeting, each containing 39 yards, at $.37½ a yard? *$102.37½.*

15. How many cubic feet in a plank 16 ft. long, 1 ft. wide, and .25 ft. thick?

16. A roadway 500 ft. long, and 16 ft. wide, is to be covered 1.5 ft. deep with gravel. How many cu. yd. of gravel will be required? *444 cu. yd. 4 cu. ft.*

17. Two men start from the same place at the same time, and travel, one 43.82 miles, and the other 34.57 miles a day. How far apart will they be at the end of 7.5 days, if they travel in opposite directions? *587.925 miles.*

18. How far apart will they be in the same time, if they travel in the same direction? *69.375 miles.*

19. How many pieces of wall-paper, each 9 yd. long and ½ yd. wide, will be required to paper 81 sq. yd. of wall? *18.*

20. A regiment was mustered into the army with 938 men. During the term of service, 93 men were killed, 76 died of sickness, 214 were mustered out, 295 were taken prisoners, 183 deserted, and 349 new recruits joined the regiment. How many men belonged to the regiment when its term of service expired? *426.*

21. If 1.25 lb. of rags are required for 1 yd. of rag carpet, how many yards of carpet can be made from 30 lb. of rags?

22. How many miles of telegraph line can be constructed with 6,116 poles, if 22 poles are set to the mile?

23. At the beginning of the year, the population of a certain city was 31,675. During the year the number of deaths was 764, and the number of births 803; 1,236 people removed from the city, and 1,394 removed to it. What was the population at the end of the year? *31,872.*

24. A freight train of 13 cars is loaded with 96 barrels of flour to each car. How many barrels of flour on the train? How many tons? *122.304 tons.*

25. An ice dealer delivered to his customers 1,296 lb. of ice daily for 27 days, 1,794 lb. daily for 26 days, 2,146 lb. daily for 56 days, 1,834 lb. daily for 24 days, and 1,310 lb. daily for 21 days. How many lbs. of ice did he deliver?

26. How many tons of ice did he deliver? *136.669.*

27. How many pounds was his average daily delivery? *1,774$\frac{74}{77}$.*

28. In 13,427 pints of beans, are how many bushels?

29. Into how many building lots can 12 acres of land be divided, each lot being 4 rods front and 10 rods deep?

30. How many bricks can be put into a pile 12 ft. long, 6 ft. wide, and 4 ft. high, each brick being 8 in. long, 4 in. wide, and 2 in. thick? (See Manual, page 220.) *7,776.*

31. Reduce 204,080 cu. in. to higher denominations. *4 cu. yd. 10 cu. ft. 176 cu. in.*

32. Reduce 11 mi. 4 yd. 2 ft. to feet. *58,094 ft.*

33. If a load of hay with the wagon weighs 3,165 pounds, and the wagon alone weighs 1,249 pounds, how much is the hay worth, at $11 a ton? *$10.538.*

34. A grocer paid $60.75 for oranges, at $6.75 a box. How many boxes did he buy?

35. A paper manufacturer paid $231.40 for rags, at $65 a ton. How many rags did he buy? *3.56 tons.*

36. A barrel inspector examined 400 barrels, and condemned 2$\frac{1}{2}$% of them. How many barrels bore inspection? *390.*

37. A provision dealer bought pork at $12.50 a hundred, and sold it at an advance of 20%. At what price did he sell it? *$15 a hundred.*

38. If I sell railroad stock which cost me $2,500, at a loss of 8$\frac{1}{2}$%, how much do I receive for it? *$2287.50.*

39. How much will it cost to insure a factory for $28,000, at 2$\frac{3}{8}$% premium?

40. The owners of the brig, Ivanhoe, paid $1\frac{1}{8}\%$ for an insurance of $17,750 for a single voyage to the West Indies. How much did the insurance cost them? *$199.68¾.*

41. How much must be paid for an insurance of $8,650 on a cargo of wheat from Milwaukee to Oswego, at $\frac{5}{8}\%$? *$54.06¼.*

42. A dairyman sent 4,320 lb. of butter to a commission-merchant, whose rate of commission was 4% on his sales. He sold the butter for $.37½ a pound. How much did the dairyman realize? *$1555.20.*

43. An agent receives 40% commission for selling maps. How much will his commission be on sales amounting to $3,280, and how much will the map publisher receive? *$1,312; $1,968.*

44. What is the interest of $341.08 for 3 yr. 10 mo., at $5\frac{1}{2}\%$? *$71.91.*

45. To how much will $74.18 amount in 2 yr. 2 mo., at 9%? *$8.64½.*

46. Find the amount of $250 for 5 yr. 7 mo., at 6%?

47. The owners of a vessel that was overdue from Liverpool, fearing that she was lost, paid 18% for an insurance of $32,000 upon her. How much did the insurance cost them?

48. A gentleman whose house cost him $18,000, had it insured for $14,000, at $\frac{7}{8}\%$ premium. Should the house burn down, what would be his entire loss? *$4122.50.*

49. How many cubic feet in a pile of 20 planks, each 12 ft. long, 10 in. wide, and 2 in. thick? *33⅓.*

50. A farmer raised 8.5 acres of flax, which yielded 850 pounds to the acre. How much was the crop worth, at $.06 a pound? *$433.50.*

51. How many acres are there in 100 miles of a road 4 rods wide? *800.*

52. In a box containing 50 sq. ft. of window-glass how many panes are there, each pane being 10 in. long and 8 in. wide? *90.*

53. If 5 cows eat $2\frac{3}{16}$ T. of hay in 5 wk., how much hay will 1 cow eat in 1 wk.? *175 lb.*

54. How many yards of carpeting will it take to cover the floor of a parlor $6\frac{2}{3}$ yd. long and $5\frac{1}{3}$ yd. wide? *$35\frac{5}{9}$.*

55. Reduce $7\frac{19}{25}$ to a mixed number.

56. A wood-chopper cut three trees into cordwood. The first tree made $3\frac{3}{4}$ cd., the second $4\frac{3}{16}$, and the third $5\frac{1}{8}$ cd. How much wood did the three trees make?

57. What will be the cost of 850 handbills, at the rate of $3.50 for the first hundred, and $1.25 for each succeeding hundred? *$12.87\frac{1}{2}$.*

58. Bought 18,280 ft. of pine flooring, at $24 a thousand. How much did it cost? *$438.72.*

59. A bushel contains 2150.42 cubic inches. How many cubic inches in 87.5 bushels? *188161.75.*

60. How many bushels in 135476.46 cubic inches?

61. A farmer has a bin 7 ft. long, 6 ft. wide, and 4 ft. deep. How many bushels will it hold? *135 bu., nearly.*

62. A gallon liquid measure contains 231 cubic inches. How many cubic inches in 275 gallons?

63. How many gallons in 13051.5 cubic inches? *56.5.*

64. A box 11 in. long, 7 in. wide, and 3 in. deep, will hold how many gallons?

65. What is the capacity in gallons of a cistern 8 ft. long, 8 ft. wide, and 7 ft. deep? *3351$\frac{3}{7}$.*

66. A tanner has a vat that will hold 1075.21 gallons. How many bushels will it hold? *117.5.*

67. At $2\frac{1}{2}\%$ commission, how much will a commission-merchant receive for selling 750 barrels of pork, at $21 a barrel?

68. A traveling agent sold 1,600 young apple-trees, at $16 a hundred. How much did his commission amount to, at 25%? *$64.*

69. How much will $290 amount to in 3 years, at 8% interest?

70. What is the interest of $2,750 for 8 mo., at 7.3%?

71. At 7%, how much interest must I pay for the use of $195.75, from May 13 to November 8 following? *$6.66.*

72. A seamstress bought a sewing-machine for $75, paying $15 down, and the balance in 4 months, with interest at 7%. What was the amount of the last payment? *$61.40.*

73. A note for $417.12, dated Jan. 10, 1866, was paid Dec. 14, 1867, with 6% interest. What was the amount paid?

74. A music dealer sold a piano, which cost him $324, for $33\frac{1}{3}$% above cost. How much did he get for it? *$432.*

75. If 23 lb. of starch can be made from 1 bu. of corn, how much corn will be required for 63,365 lb. of starch?

76. A railroad company having 343 A. of woodland, cut from 28.5 A. 58.75 cd. of wood per acre, from 93.3 A. 52.5 cd. per acre, and from 112.7 A. 48.25 cd. per acre. How much wood was cut, and how many acres of wood remained standing?

12010.4 cd.; 108.5 A.

77. A farmer sowed $13\frac{1}{3}$ bu. of wheat on $10\frac{1}{2}$ acres of land, and the yield was $14\frac{1}{2}$ bu. per acre. How much wheat did he get above his seed? *$139\frac{1}{3}$ bu.*

78. How many tons will 500 barrels of flour weigh? *49.*

79. 23 teams were employed 47 days in drawing earth for a railroad embankment, each team averaging 15 cu. yd. 24 cu. ft. a day. How much earth was in the embankment?

17175 cu. yd. 24 cu. ft.

80. How much can I save in a year, if I earn $100 a month for 10 months, and spend $68.63 every month? *$176.44.*

81. How many feet in a board 16 ft. long and 9 in. wide?

82. A manufacturer finds that merino wool loses .42 of its weight in cleaning. How much weight will 2345.5 pounds lose? *985.11 pounds.*

83. Find the interest of $25 for 7 yr. 3 mo. 24 da., at 10%.

$18.29.

84. What is the amount of $110.62 for 3 yr. 7 mo. 28 da., at 7%? *$138.97.*

85. What sum must be paid to cancel a debt of $219.16, which has been due 1 yr. 6 mo. 14 da., at the rate of interest in this State?

86. I bought a house for $1,850, and afterward sold it at an advance of 30%. How much was my gain, and for how much did I sell it? *Gain, $555; selling price, $2,405.*

87. The sum of three parts is 100 mi., and two of the parts are 33 mi. 225 rd. 2 yd. 2 ft., and 17 mi. 90 rd. 3 yd. What is the third part?

88. $13\frac{1}{2} + \frac{4}{9} + 5\frac{3}{8} + \frac{5}{6} +$ what number $= 47\frac{5}{12}$? *27 1/2.*

89. After traveling 21% and 18% of a journey of 425 miles, what % of the journey had I yet to travel? How many miles had I yet to travel? *259.25 miles.*

90. How much will $\frac{37}{40}$ of a ton of hay cost, at $7.50 a ton?

91. Brass is composed of copper and zinc. In a quantity of brass that weighed $\frac{21}{32}$ of a ton, the copper weighed $\frac{7}{16}$ of a ton. What was the weight of the zinc? $\frac{7}{32}$ *T.*

92. If a family burn $\frac{25}{28}$ of a cord of wood in 30 days, how much wood will they burn in 1 day? $\frac{5}{168}$ *cd.*

93. From what number must I subtract 984,006 to obtain a remainder of 9,276,985? *10,260,991.*

94. The subtrahend is six thousand and twenty-four ten-thousanths, and the remainder four thousand ninety-six hundred-thousandths. What is the minuend?

95. The minuend is 17 cu. yd., and the remainder 16 cu. yd. 1,596 cu. in. What is the subtrahend? *26 cu. ft. 132 cu. in.*

96. If 16 shoemakers make 520 pairs of shoes in 16.25 days, how many pairs can 1 workman make in 1 day? *2.*

97. A farmer exchanged 85 lb. of butter at $.21 a pound, for flannel at $.35 a yard. How many yards did he receive?

98. If a young man smokes 1,284 cigars in a year, how much will his year's supply cost him, at $28.50 a thousand? *$36.59.*

99. What will be the cost of 17,890 feet of hemlock lumber, at $11 a thousand?

100. What number, multiplied by 7,296, will produce 292,518,528?

101. The multiplicand is $19\frac{1}{9}$, and the product $319\frac{3}{7}$. What is the multiplier? *16 5/7.*

102. The product of three factors is 29.2923, and two of the factors are 11.4 and 285.5. What is the third factor?

103. How many square inches in a mirror 5 ft. 3 in. high and 26 in. wide?

104. In a village lot 66 ft. front by 132 ft. deep, are how many sq. rd.? *32.*

105. If 16 men in 17.25 days can mine 529.92 tons of iron ore, how much ore can 1 man mine in 1 day? *1.92 T.*

106. A clergyman had his household furniture insured for $850, and his library for $650, at $\frac{3}{4}\%$. What premium did he pay annually? *$11.25.*

107. The dividend is 9, and the quotient 576. What is the divisor? *.015625.*

108. What number divided by $19\frac{6}{7}$ will give a quotient of $1\frac{43}{121}$?

109. A farmer cures his hams by the following recipe: For every 100 lb. of meat, 9 lb. of salt, 5 oz. of saltpeter, 4 oz. of ground pepper, and 1 qt. of molasses. What quantity of each ingredient must he use for 675 pounds of meat?

Salt, 60.75 lb.; saltpeter, 33.75 oz.; ground pepper, 27 oz.; molasses, 6.75 qt.

110. In salting beef, the same farmer uses the following recipe: For every 100 lb. of meat, 6 qt. of salt, 1 qt. of molasses, and 4 oz. of saltpeter. What quantity of each ingredient must he use for 380 lb. of meat?

Salt, 22.8 qt.; molasses, 3.8 qt.; saltpeter, 15.2 oz.

111. Which is the more advantageous, to borrow $175 at 7% interest to pay house-rent in advance, or to pay $200 rent at the end of the year? How much the more advantageous?

To borrow, by $12.75.

112. A man can hire a farm of 97 acres for $500 per annum, or he can buy it for $70 an acre. If money is worth 6%, which is the cheaper course, and how much the cheaper?

To buy the farm, by $92.60 per annum.

113. A farmer bought 80 sheep, at $4.20 a head, giving his note, payable in 6 mo., at 7%. At the end of the 6 mo. he sold the sheep at $5.25 a head cash, and paid the note. How much did he get for the keeping of the sheep? *$72.24.*

114. A man had his life insured for $5,000 at the age of 29 years, and died at the age of 47. His yearly premiums averaged $71.85. How much more did his family receive than he had paid?

MANUAL

OF

METHODS AND SUGGESTIONS.

A Word with Teachers.—This Manual is intended to give you hints and suggestions rather than detailed methods of instruction; and to call your attention to those points which require your special efforts, if you would secure that thoroughness in your pupils which is essential to real progress. It is not intended to lay down prescribed forms for you to follow, but to give you such hints and suggestions as will enable you to work out details of methods of instruction, and to adapt them to your classes, in accordance with your methods of thought.

Use of Objects.—Children gain ideas more readily by perception than by reflection. You should, therefore, illustrate the subjects of lessons by *Visible Objects* whenever this is practicable. For example:

In Notation.—A child may be aided in comprehending the idea of a ten and of a hundred by the use of counters. Take a quantity of beans, or other suitable objects, make a pile of ten of them for *a ten*, and ten such piles near together for ten tens or *a hundred*.

In Addition.—Illustrate the addition of two numbers, as 37 and 48, thus: 3 piles of tens and 7 counters or ones may represent 37; and 4 piles of tens and 8 counters or ones may represent 48. Then putting the 7 counters and 8 counters together, there will be enough counters for 1 ten and 5 counters or ones more: The 1 ten, 3 tens, and 4 tens, together, are 8 tens, and the 8 tens and 5 ones, are 85.

In Subtraction.—The process may be illustrated in a similar manner.

In Compound Numbers.—Let the pupils see and handle the various measures, and they will get clear ideas of a quart, a bushel, a foot, a pound, or any other denomination. Let them measure water, and see that 2 pints are 1 quart, and 4 quarts 1 gallon. Let them exercise their judgment upon the capacity of vessels, by estimating how much a pail, a pan, a pitcher, a cup, or a bottle, will hold; and then require them to measure the vessel to correct their judgment. In the same manner let them measure sand, or corn, to become familiar with the denominations of dry measure. Also, let them by trial see if a quart liquid measure is the same as a quart dry measure, etc.

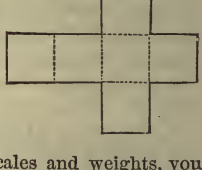
To make them familiar with the denominations of distance, let them draw lines, an inch, a foot, and a yard long, upon the blackboard: judge of the width, length, and height of the room, or the

house; and then measure the various distances to correct their judgment. Again, for greater distances, let them measure the distance of a rod along the fence, then 40 rods or 1 eighth of a mile on the road. Let them estimate the distances between different objects in the room, about the yard, and along the road, and afterward measure them.

If no other measures can be obtained, cut from a lath, or any other straight stick, a piece 1 inch long; another 1 foot long, and mark it off into inches with a knife or pencil; and another 1 yard long, and mark it off into feet. A cord or a piece of rope may be used for the measure of a rod.

In square measure, have a square inch, a square foot, and a square yard. You can show that these might be used to measure with, but that it is more convenient to take dimensions with linear measures.

The measure of a cubic inch may be made by cutting out a piece of pasteboard in this form, cutting about one half through the thickness where the lines cross it, folding it together, and fastening the joining edges. Larger cubes may be made in the same manner. A box 1 foot long, 1 foot wide, and 1 foot high, is a measure of a cubic foot. The surface should be marked off into square inches.



If your schoolroom is not supplied with scales and weights, you can prepare a few weights by making packages or bags of shot, sand, or pebbles, weighing 1 oz., 1 fourth lb., 1 half lb., 1 lb., 5 lb., and 10 lb. With these and other objects you can exercise pupils until they can judge of weight with considerable accuracy.

ORIGINAL ILLUSTRATIONS.—In many cases you will interest your pupils, as well as assist them to comprehend a problem, by drawing figures or diagrams upon the blackboard.

Tables of Combinations.—Require the pupils to construct the Tables of Addition, Subtraction, Multiplication, and Division, before committing them to memory. They can do this by using counters to form the combinations. In this way they will more fully comprehend the meaning of the tables, and also prove them to be correct.

Only a small part of any table should be assigned for one lesson, and upon that the pupils should be made thorough before an advanced lesson is assigned. Let them from memory write that part of the table learned upon their slates or paper, and upon the blackboard. Also yourself write upon the blackboard portions of the table, not in regular order, and without the results, and require the pupils to give the results without hesitation. For example, take the 4's in the Addition Table, and writing them as directed,

4	*4*	*4*	*4*	*4*	*4*	*4*	*4*	*4*	*4*
4			*8*	*3*	*9*	*5*	*0*		*7*

point to the numbers forming any combination, and require instant answers. This may be done first with the whole class, and then with each pupil separately. Pursue a similar course with review exercises in all the tables. Also, give numerous problems to the class, and require instantaneous answers; and let pupils give original problems to each other for mental solution.

Inductive and Oral Exercises.—"Make haste slowly" in passing over these portions of the book. Exercise pupils upon the definitions and signs until they are perfectly familiar with them. The *Oral Exercises* should be assigned to the class in connection with the *Tables of Combinations*. For example, when the class have learned the addition of 3's from the table, assign the oral exercise in counting by 3's. The forms given for the first two or three exercises under each number should be continued through all the exercises. Give your pupils frequent practice upon these exercises. Let the school together recite from them 5 minutes at a time, or when they are passing into and out of the room, or when classes are moving.

All the possible combinations of the numbers used are given in the oral exercises. If pupils are ready in these, they will be ready and accurate in computations.

Conducting Recitations.—In conducting recitations, never forget that the ends to be accomplished are fourfold, viz.:

1*st*. *To impart new and valuable instruction*, adapted in kind and amount to the condition of the minds of your pupils;

2*d*. *To teach pupils to think*, by so guiding their inquiries that they shall discover truths for themselves;

3*d*. *To make them thorough*, by always requiring accurate recitations and explanations; and,

4*th*. *To keep them interested* in their studies.

The following order in conducting recitations has been found to secure these results:

1*st*. Hear as many of the class recite the lesson assigned as time will permit, requiring them to go through the recitation without interruption from other members of the class, and with as little prompting and as few questions as possible from you. Throw no stumbling-blocks in their way at this time; for pupils who recite a new lesson well, do all you have a right to ask of them at first.

2*d*. After this, test their knowledge of the lesson, by fair but critical questions. In this way you will find what instruction they need.

3*d*. Impart the needed instruction and no more, always observing this rule: "*Never tell a child anything you wish him to remember, without requiring him to tell it to you again.*"

4*th*. Make practical applications of the lesson.

5*th*. Review such portions of previous lessons as you deem important.

To express in words what we have learned, is to make the knowledge our own, to make it more clear to our understanding, and to fix it in our memory. Therefore, require pupils to write out upon their slates or paper a full explanation of one or more of the problems in the lesson, before coming to recitation, and also upon the blackboard at recitation. The explanations of examples solved in this book may be taken as models; but as many of these, especially in integers, are more minute than you should require for the solution of the problems, a few hints are here given that may aid you in properly directing your pupils in their explanations.

ADDITION.—Suppose you have assigned a solution and explanation of problem 81, page 29, to be written out by a pupil. His explanation may be as follows:

The farmer raised as many bushels of grain as the sum of 587 *bushels*, 1,229 *bushels*, 643 *bushels*, 184 *bushels*, 259 *bushels, and* 296 *bushels.*

Since only figures occupying the same place in different numbers can be added, I wrote the given numbers (or *the given parts*) *with ones under ones, tens under tens, and so on.*

I then commenced with the ones, and added each column separately, writing the right-hand figure of each sum under the column added, and adding the left-hand figure with the next column.

The sum, 3,198, *is the total number of bushels of grain which the farmer raised.*

SUBTRACTION. — For example, take problem 57, page 41. The pupil's explanation may be this:

The hay weighed as much as the difference between 2,656 *pounds and* 987 *pounds.*

Since only figures occupying the same place in different numbers can be subtracted the one from the other, I wrote the subtrahend below the minuend, placing ones under ones, tens under tens, and so on.

I then commenced with the ones, and subtracted each figure of the subtrahend from the corresponding figure of the minuend, and wrote the result directly below in the remainder.

The remainder, 1,669, *is the number of pounds of hay in the load.*

You may require the pupil to embrace, in his explanation, the process when any figure of the subtrahend exceeds the corresponding figure of the minuend, until you are sure he is familiar with it; afterward it may be omitted.

MULTIPLICATION.—Take problem 107, page 61. The explanation of the solution may be as follows:

The cost of manufacturing 360 *pianos was* 360 *times as much as the cost of manufacturing* 1 *piano.*

I therefore multiplied $270, *the cost of manufacturing* 1 *piano, by* 360, *and I obtained* $97,200, *the total cost.*

You may require the pupil to explain the process—writing the factors, multiplying, adding partial products, and annexing ciphers for

the final product—until he is familiar with all the steps; after which an explanation like the above is sufficient.

DIVISION.—First Form: both terms like denominations.—Take problem 37, page 74.

Since \$6 *was the price of* 1 *cord of wood, for* \$6,828 *as many cords were bought as the number of times* \$6 *is contained in* \$6,828, *which is* 1,138 *times.*

Second Form: the divisor an abstract number.—Take problem 51, page 76.

He received 1 *twelfth as much for building* 1 *rod of fence as he did for building* 12 *rods.*

I therefore divided \$156, *the cost of building* 12 *rods, by* 12, *and the result,* \$13, *is the cost of building* 1 *rod.*

You may require the pupil to explain all the steps in the process of Division until he is familiar with them; after which the above form of explanation is sufficiently full.

The above five explanations may be modified to meet the conditions of any problem in integers, decimals, compound numbers, percentage, and for most problems in Fractions:

FRACTIONS.—The two following explanations may be of some aid in Multiplication and Division of Fractions.

Problem 17, page 176.—*Seven eighths of any number is* 7 *times as much as one eighth of the number; and one eighth of any number is found by dividing it by* 8.

I therefore divided 300 *pounds by* 8 *to find* 1 *eighth of them, and then multiplied the quotient,* 37½, *by* 7, *to find* 7 *times* 1 *eighth, or* 7 *eighths of them.*

The result, 262½, *is the number of pounds used.*

Problem 22, page 181.—*Since* \$$\frac{3}{10}$ *is the price of* 1 *yard of ribbon, for* \$¼ *as many yards can be bought as the number of times* \$$\frac{3}{10}$ *is contained in* \$¼.

\$$\frac{3}{10}$ *is contained in* \$¼ *as many times as* 10 *times* \$$\frac{3}{10}$, *or* \$3, *is contained in* 10 *times* \$¼, *or* \$$\frac{10}{4}$; *and* \$3 *is contained in* \$$\frac{10}{4}$, $\frac{10}{4}$ *or* 2½ *times.*

Therefore, 2½ *yards is the required result.*

COMBINATIONS.—Rapidity in computation is an acquirement much to be desired by all. Stimulate pupils to pronounce results rapidly, and without naming the numbers combined. Thus,

In Addition—Instead of allowing them to say, 7 and 5 are 12, 12 and 6 are 18, 18 and 9 are 27, they should be taught to pronounce the partial results orally, as they point to each figure added; thus, 7, 12, 18, 27.

9
6
5
7

In Subtraction—The usual manner is this: 2 from 6 leave 4; 3 from 10 leave 7; 5 and 1 are 6, and 6 from 14 leave 8; 1 and 1 are 2, and 2 from 3 leave 1. Teach pupils to perform all the combinations mentally, and to pronounce the partial results orally; thus, looking at the 6 and 2, the pupil says 4; looking at the 0 and 3, he says 7; looking at the 4 and 5, he says 8; and looking at the 3 and 1, he says 1.

3406
1532
1874

In Multiplication—Teach him as each figure of the multiplicand is reached, to pronounce the product and the sum; thus, 35; 28, 31; 21, 24; 56, 58.

$$\begin{array}{r} 8345 \\ \times\quad 7 \\ \hline 58415 \end{array}$$

In Short Division—Let him name only the quotient figure and the remainder; thus, 2 and 3 over, 8 and 1 over, 4, 1.

$$11364 \div 4 = 2841$$

Require pupils to study the solutions and explanations, and to state the principles upon which any given method is based. No rule should be assigned as a lesson until the principles involved are clearly understood.

Pupils should commit to memory the definitions, principles, and other matter in Italics, but in all cases they should be required to show that they understand what they have memorized.

If you find that pupils 'work to get the answers' given in the book, rather than to understand and apply principles and methods, change one or more figures in the problems, when you assign a lesson, and the printed answers will be of no help to the pupils.

Many of the Review Problems in each chapter may be solved in more than one way. Call out the different methods from the class, and thus stimulate them to *think:* and after the solutions have been presented, exercise their judgment by requiring them to state which of the solutions presented is the best, and why.

We will now pass to the suggestions to which references are made in various parts of the book.

Page 10.—Exercise pupils in writing and reading all the numbers to 100. Require them to write the numbers given in the Exercises, in columns, neatly, placing ones under ones, and tens under tens, and to bring their work to recitation for inspection.

Page 11.—Drill the class in writing and reading numbers of three figures until they make no mistakes. Then require them to write the exercises given on page 12, first upon their slates, and afterward upon the blackboard. See that their work is neatly done, the figures well made, and ones written under ones, tens under tens, etc.

Page 13.—Practice pupils upon the two periods, until the places and their names are familiar to them. You will aid them in this, by writing upon the blackboard two periods of ciphers, and under these require the pupils to write the exercises, and also to name the place occupied by each cipher.

Page 15.—To enable pupils to write and read numbers readily, they should be able to tell promptly how many figures are required to express any given number less than 1,000,000,000. To secure this ability on their part, you can frequently question them in this manner: How many figures express ones? Thousands? Millions? etc.

How many figures are required to express 50 thousand? How many to express 50 thousand 7 hundred? To express 50 thousand 38? etc.

How great a number can be expressed by four figures? By seven figures? By five figures? etc.

How many figures are required to express ten-thousands? How many to express hundreds? Hundred-millions? etc.

Require pupils to point off all whole numbers into periods of three figures each, commencing at the right.

Explain to them that, in reading numbers, they must always commence at the left, and read each period by itself as a distinct number, pronouncing after it the name of the period.

Before assigning the Review Exercises on page 16, drill the class both in reading and writing numbers: first, upon numbers containing no ciphers; next, upon numbers containing one cipher,—the cipher occupying a different place in each number; then upon numbers containing two ciphers in all possible places, both together and separate; then upon numbers containing three ciphers, and so on.

The Review Exercises (page 16) are test exercises; and pupils should be able to write all of them before passing to the next section.

Page 17.—See suggestions on *Inductive and Oral Exercises*, page 211.

Page 19.—See suggestions on *Tables of Combinations*, page 210.

Page 20.—See suggestions on *Tables of Combinations*, page 210.

Page 22.—Explain to pupils that, since we can only add figures occupying the same place in different numbers, and since it is more convenient to have the figures to be added stand in a column, we write the parts, for convenience, ones under ones, tens under tens, etc. Also explain that we commence at the right to add, not from necessity,—for we may commence at any other place,—but because it is more convenient.

In order that your pupils may do their work methodically, require them uniformly to commence either at the top or at the bottom of columns of figures to add them.

Page 24.—Explain the terms *parallel* and *horizontal*, so that your pupils will clearly understand their meaning.

Page 25.—The signs for wood-land, meadow, fences, streams, etc., on this map are the conventional signs used by topographers and surveyors. They always mean the same thing whenever found upon a properly drawn map.

PROBLEM 41.—Be sure that the pupil discovers that there are two fences, each 47 rods long, and two others each 34 rods long.

Page 26.—If more problems are required, a large number may be formed from this table of railroad distances. Thus, assign for a lesson

to find the distance from Worcester to each of the other places named in the table.

Page 27.—Require the pupil to add an example, first by the method given on page 24, and then by the method here given. Then require him to compare the results, and thus to discover that the latter method is only an abridgment of the former.

Page 31.— See suggestions on *Inductive and Oral Exercises*, page 211.

Page 32.—See suggestions on *Tables of Combinations*, page 210.

Page 33.—See suggestions on *Oral Exercises*, page 211.

Page 35.—If Principle II. is not understood by your pupils, lead them to see the truth of it, by questions. Thus: What is the difference between $7 and $3? Between 7 boys and 3 boys? Between 7 tens and 3 tens? Between 7 thousands and 3 thousands? etc.

Page 37.—The method of adding 10 to both minuend and subtrahend before subtracting, when any figure in the subtrahend exceeds the corresponding figure in the minuend, is thought to be too difficult for quite young beginners, and hence it is not introduced into this book. Those teachers who prefer to use it, will, of course, do so.

By the aid of objects show the class that 1 ten and 6 ones are 16 ones, and that 7 tens and 6 ones are 6 tens and 16 ones.

Page 40.—Require the pupil to solve this example first by the method given on page 37, and then by the method here given. He will thus see that the latter method is only a abridged form of the former.

Page 41.—Require your pupils uniformly to call the next left-hand figure of the minuend 1 less, or the next left-hand figure of the subtrahend 1 more.

Page 45.— See suggestions on *Inductive and Oral Exercises*, page 211.

Page 47.—See suggestions on *Tables of Combinations*, page 210.

Page 48.— See suggestions on *Inductive and Oral Exercises*, page 211.

Page 49.—Very few learners fully comprehend the fact that every problem in multiplication can be solved by addition. In order to fix this fact firmly in the minds of your pupils, require them to solve the first 10 problems on page 50, both by addition and multiplication.

Page 52.—Require pupils to speak of the true multiplicand only as the multiplicand. For example, in finding 345 times 7, they may multiply 345 by 7; but in the explanation of their solution, require them to say 345 times 7.

Page 61.—If this explanation is not fully understood, solve the example in different ways, and compare the results. Thus, you may use either of the following solutions:

67000	67000
2100	21
67000	67
134000	134
140700000	1407000
	100
	140700000

But, in practice, never permit pupils to write the factors for multiplication as shown in the first of these solutions.

Page 62.—First Reference.—Be sure to give your pupils clear ideas of the terms *factors* and *parts*. Thus, 7, 5, and 2 are factors of 70, but they are parts of 7 + 5 + 2, or 14, and also of 752.

Second Reference.—Round trip, out and back again, or twice over the road, once each way. This is also called *doubling the road.*

Third Reference.—See suggestions on *Conducting Recitations*, page 211.

Page 64.—See suggestions on *Inductive Exercises*, page 211.

Page 65.—Illustrate, by the use of objects, fractional division, or the division of a given number of things into a certain number of equal parts. For example, with a quantity of beans or corn, or a large number of other counters before the class, divide the number of objects into 2, 3, 4, 5, etc., equal parts, and then require each pupil to do the same. Also, ask numerous questions like these: How can you find 1 half of these objects? How 1 third of them? How 1 fifth of them? 1 fifteenth? 1 fortieth? etc. How can you find 1 third of any number? How 1 eighth of it? How 1 twenty-fourth? 1 seventy-first? 1 ninetieth? etc.

Page 66.—See suggestions on *Tables of Combinations*, page 210.

Page 67.—See suggestions on *Oral Exercises*, page 211.

Page 73.—Require the class now to solve the problems on pages 70, 71, 72, and 73, by short division.

Page 75.—Do not permit pupils to leave these principles until they thoroughly comprehend them. You can test their understanding by placing erroneous processes upon the blackboard, and requiring them to point out the errors, and to tell why they are wrong. Thus, place upon the blackboard this example, and require pupils to correct it, and to give reasons for the correction:

12684 | 28	
112	4413
148	
112	
36	
28	
84	
84	

Page 98.—If your pupils are slow to learn the notation of decimals, you may find the following direction of value to them:

To Write any Decimal Number,

Write it first as an integer, and then place the decimal point according to the table of values of decimal numbers.

Exercise your pupils until they are familiar with this table. Ask such questions as the following, until they can give correct answers promptly:

How many decimal figures express thousandths? How many hundred-thousandths? Hundredths? Ten-millionths? etc.

What decimal is expressed by four decimal figures? By three figures? By eight figures? etc.

Page 99.—Pupils should be thoroughly grounded in this table, as a knowledge of it will enable them to master the *theory* of computations in decimals with increased facility. For a test of their knowledge, write a line of 1's on the blackboard,

11111111.11111111,

and, commencing at the right, require your pupils to give the law of increase toward the left, first to the decimal point, and afterward to hundred-millions; thus, 10 hundred-millionths are 1 ten-millionth, 10 ten-millionths are 1 millionth, 10 millionths are 1 hundred-thousandth, and so on, to 10 ten-millions are 1 hundred-million.

Then, commencing at the left-hand figure of the integer, require them to give the law of decrease to the right; thus: 1 hundred-million is 10 ten-millions, 1 ten-million is 10 millions, 1 million is 10 hundred-thousands, ... 1 one is 10 tenths, 1 tenth is 10 hundredths, 1 hundredth is 10 thousandths, ... 1 ten-millionth is 10 hundred-millionths.

After the class can run through this number with facility, write lines of 4's, 7's, 3's, and so on, and practice them upon the numbers thus formed in the manner above directed.

Page 100.—First Reference.—Illustrate these principles by numerous examples like these; show that .5 = .50 = .500 = .5000; that .75 = .750 = .75000, and so on; also, that annexing a cipher changes the values of the figures of an integer, by removing *ones* to *tens*, *tens* to *hundreds*, etc.

Second Reference.—In reading numbers, the word *and* should be used only between the integral and decimal parts of mixed numbers. If 441 be read four hundred and forty-one, certain entirely different numbers would be read exactly alike. Thus 400.041 and .441 would both be read four hundred and forty-one thousandths.

The following unlike numbers would be read alike: 15000.00008 and .15008; 5000.0024 and .5024; 964000.000072 and .964072.

Ten-thousand, hundred-thousand, ten-thousandths, hundred-thousandths, etc., should be written as compound words; otherwise, *three hundred ten thousandths* might mean .310 or .0300.

The exercises on page 100 will test the ability of pupils to write and read decimals. For additional practice, write decimal numbers upon the blackboard for your pupils to read, and require them to write numbers from dictation, until they can write and read decimals with the same facility as integers.

Page 101.—Explain the difference between Addition of Decimals and Addition of Integers. Do the same with Subtraction, Multiplication, and Division.

Page 129.—Be sure that pupils always write abbreviations correctly, and in their proper places, *i. e.*, that they write the proper abbreviated form, and place a period after it; and that the abbreviation be placed after the number, except the sign of dollars ($), which should always be placed before the number.

Before your pupils take up the tables in Compound Numbers, read carefully the suggestions on *Use of Objects*, page 209.

Page 146.—Make your pupils so familiar with this table, that they can tell at once the number of any month, and the number of days in it.

Page 147.—In the table of weight (see **185**), the denomination *grain* is not named, as it is not used in commercial weight. Explain to your pupils that if a pound is divided into 7000 equal parts, 1 of these parts is 1 grain: and hence, *a grain is* 1 *seven-thousandth of a pound; a gram is* 15432 *seven-millionths of a pound;* and *a pound contains a little more than* 453.6 *grams.*

Page 165.—Thorough drills upon these principles are indispensable to the further intelligent progress of the pupil. Therefore, be sure that he understands them before passing to Section II.

Page 169.—The common solution is similar to Reduction Ascending, and it may be explained in the same manner, by regarding the numerator as a number to be reduced to a higher denomination, and the denominator as that number of the given denomination that equals one of the required higher denomination. Thus, the process of reducing 17 fourths to a mixed number is the same as that of reducing 17 pecks to bushels, or 17 quarts to gallons.

Page 170.—The common solution is similar to Reduction Descending, and may be explained in the same manner, by regarding the integer as the higher denomination, and the numerator of the fraction as the number of the next lower denomination. Thus, the process of reducing $5\frac{3}{4}$ to fourths is the same as that of reducing 5 bu. 3 pk. to pk., or 5 gal. 3 qt. to qt.

Page 172.—Fix in the minds of pupils the fact that numerators only are added or subtracted, and denominators are written to give name or denomination, the same as in compound numbers. Give numerous illustrations, such as 4 men + 3 men + 5 men; $4 + $3 + $5; 4 parts + 3 parts + 5 parts; 4 ninths + 3 ninths + 5 ninths, or $\frac{4}{\text{ninths}} + \frac{3}{\text{ninths}} + \frac{5}{\text{ninths}}$; 13 pens — 8 pens; $13 — $8; 13 parts — 8 parts; 13 ninths — 8 ninths, or $\frac{13}{\text{ninths}} - \frac{8}{\text{ninths}}$, etc.

Page 182.—The method "Invert the divisor and proceed as in multiplication," is introduced in Cancellation, p. 184. Therefore pupils should be required to solve and explain these problems in the manner here shown, as this course will make them familiar with the reasons for the process, which is not likely to be the case if they at once adopt the mechanical convenience of inverting the divisor.

Page 184.—Be sure that pupils understand that a factor in either term will cancel only *one like factor* in the other term. Also, always require them to write 1 in the place of a canceled term, as pupils are liable to think that when a term is canceled, a 0 belongs in its place. Test their knowledge upon this point, by questions and examples.

Page 197.—Explain the fact that any number of months forms the numerator of a fraction, of which 12 (the number of months in a year) is the denominator; and that this multiplying fraction can often be reduced to lower terms before multiplying. Thus, 2 mo. $= \frac{2}{12} = \frac{1}{6}$ yr., 3 mo. $= \frac{3}{12} = \frac{1}{4}$ yr., 4 mo. $= \frac{4}{12} = \frac{1}{3}$ yr., 6 mo. $= \frac{6}{12} = \frac{1}{2}$ yr., 8 mo. $= \frac{8}{12} = \frac{2}{3}$ yr., 9 mo. $= \frac{9}{12} = \frac{3}{4}$ yr., 10 mo. $= \frac{10}{12} = \frac{5}{6}$ yr. A familiarity with this fact will often enable pupils to abridge their work by multiplying the interest for 1 year by the time expressed in years and fractions or decimals of a year.

Page 201.—When pupils pass over these General Review Problems, exercise their ingenuity in producing different methods for solving the same problem. Thus, problem 30, page 203, may be solved in three ways:

1*st.* Find the contents of the pile in cubic inches, and then divide them by the number of cubic inches in a brick.

2*d.* Find the cubic contents of the pile in feet, and multiply them by the number of bricks in a foot; and,

3*d.* Find how many bricks long, wide, and high, the pile is, and then multiply these numbers together. Again,

Problem 50, page 104, may be solved in two ways:

1*st.* Find the whole number of pounds raised, by multiplying the number of pounds per acre by 8.5, the number of acres, and multiply \$.06, the price of 1 pound, by this product; and,

2*d.* Find what he received for 1 acre of flax, by multiplying \$.06, the price per pound, by 850, the number of pounds raised to the acre; and then multiply this result by 8.5, the number of acres raised.

This course will teach pupils that, in many cases, there is more than one right way to solve a problem, and it will also teach them to *think methodically.*

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